

Chapter

6

Solving Linear Inequalities

What You'll Learn

- **Lessons 6-1 through 6-3** Solve linear inequalities.
- **Lesson 6-4** Solve compound inequalities and graph their solution sets.
- **Lesson 6-5** Solve absolute value equations and inequalities.
- **Lesson 6-6** Graph inequalities in the coordinate plane.

Key Vocabulary

- set-builder notation (p. 319)
- compound inequality (p. 339)
- intersection (p. 339)
- union (p. 340)
- half-plane (p. 353)

Why It's Important

Inequalities are used to represent various real-world situations in which a quantity must fall within a range of possible values. For example, figure skaters and gymnasts frequently want to know what they need to score to win a competition. That score can be represented by an inequality. *You will learn how a competitor can determine what score is needed to win in Lesson 6-1.*



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 6.

For Lessons 6-1 and 6-3

Solve Equations

Solve each equation. (For review, see Lessons 3-2, 3-4, and 3-5.)

1. $t + 31 = 84$
2. $b - 17 = 23$
3. $18 = 27 + f$
4. $d - \frac{2}{3} = \frac{1}{2}$
5. $3r - 45 = 4r$
6. $5m + 7 = 4m - 12$
7. $3y + 4 = 16$
8. $2a + 5 - 3a = 4$
9. $\frac{1}{2}k - 4 = 7$
10. $4.3b + 1.8 = 8.25$
11. $6s - 12 = 2(s + 2)$
12. $n - 3 = \frac{n + 1}{2}$

For Lesson 6-5

Evaluate Absolute Values

Find each value. (For review, see Lesson 2-1.)

13. $|-8|$
14. $|20|$
15. $|-30|$
16. $|-1.5|$
17. $|14 - 7|$
18. $|1 - 16|$
19. $|2 - 3|$
20. $|7 - 10|$

For Lesson 6-6

Graph Equations with Two Variables

Graph each equation. (For review, see Lesson 4-5.)

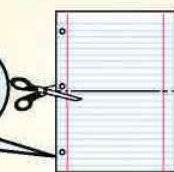
21. $2x + 2y = 6$
22. $x - 3y = -3$
23. $y = 2x - 3$
24. $y = -4$
25. $x = -\frac{1}{2}y$
26. $3x - 6 = 2y$
27. $15 = 3(x + y)$
28. $2 - x = 2y$

FOLDABLES™ Study Organizer

Make this Foldable to record information about solving linear inequalities. Begin with two sheets of notebook paper.

Step 1 Fold and Cut

Fold in half along the width. Cut along fold from edges to margin.



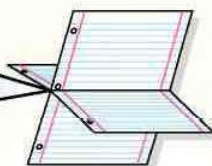
Step 2 Fold a New Paper and Cut

Fold in half along the width. Cut along fold between margins.



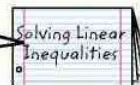
Step 3 Fold

Insert first sheet through second sheet and align folds.



Step 4 Label

Label each page with a lesson number and title.



Reading and Writing As you read and study the chapter, fill the journal with notes, diagrams, and examples of linear inequalities.

6-1

Solving Inequalities by Addition and Subtraction

What You'll Learn

- Solve linear inequalities by using addition.
- Solve linear inequalities by using subtraction.

Vocabulary

- set-builder notation

How are inequalities used to describe school sports?

In the 1999–2000 school year, more high schools offered girls' track and field than girls' volleyball.

$$14,587 > 13,426$$

If 20 schools added girls' track and field and 20 schools added girls' volleyball the next school year, there would still be more schools offering girls' track and field than schools offering girls' volleyball.

$$14,587 + 20 \quad ? \quad 13,426 + 20$$

$$14,607 > 13,446$$

USA TODAY Snapshots®



Girls gear up for high school sports

High school girls are playing sports in record numbers, almost 2.7 million in the 1999–2000 school year. Most popular girls sports by number of schools offering each program:



Source: National Federation of State High School Associations.

By Ellen J. Horrow and Alejandro Gonzalez, USA TODAY

Study Tip

Look Back

To review **inequalities**, see Lesson 1-3.

SOLVE INEQUALITIES BY ADDITION Recall that statements with greater than ($>$), less than ($<$), greater than or equal to ($\geq$), or less than or equal to ($\leq$) are **inequalities**. The sports application illustrates the **Addition Property of Inequalities**.

Key Concept

Addition Property of Inequalities

- Words** If any number is added to each side of a true inequality, the resulting inequality is also true.
- Symbols** For all numbers a , b , and c , the following are true.
 - If $a > b$, then $a + c > b + c$.
 - If $a < b$, then $a + c < b + c$.
- Example**

$$\begin{array}{rcl} 2 & < & 7 \\ 2 + 6 & < & 7 + 6 \\ 8 & < & 13 \end{array}$$

This property is also true when $>$ and $<$ are replaced with $\geq$ and $\leq$.

Example 1 Solve by Adding

Solve $t - 45 \leq 13$. Then check your solution.

$$t - 45 \leq 13 \quad \text{Original inequality}$$

$$t - 45 + 45 \leq 13 + 45 \quad \text{Add 45 to each side.}$$

$$t \leq 58 \quad \text{This means all numbers less than or equal to 58.}$$

CHECK Substitute 58, a number less than 58, and a number greater than 58.

$$\text{Let } t = 58.$$

$$58 - 45 \stackrel{?}{\leq} 13$$

$$13 \leq 13 \quad \checkmark$$

$$\text{Let } t = 50.$$

$$50 - 45 \stackrel{?}{\leq} 13$$

$$5 \leq 13 \quad \checkmark$$

$$\text{Let } t = 60.$$

$$60 - 45 \stackrel{?}{\leq} 13$$

$$15 \neq 13$$

The solution is the set {all numbers less than or equal to 58}.

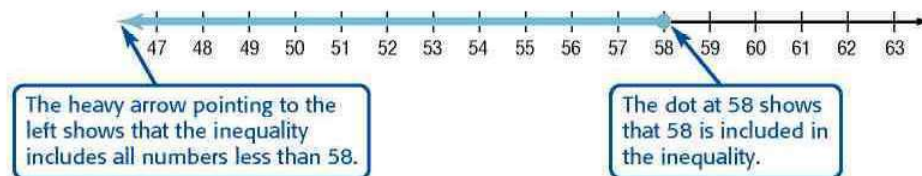
Study Tip

Reading Math

$\{t | t \leq 58\}$ is read *the set of all numbers t such that t is less than or equal to 58*.

The solution of the inequality in Example 1 was expressed as a set. A more concise way of writing a solution set is to use **set-builder notation**. The solution in set-builder notation is $\{t | t \leq 58\}$.

The solution to Example 1 can also be represented on a number line.



Example 2 Graph the Solution

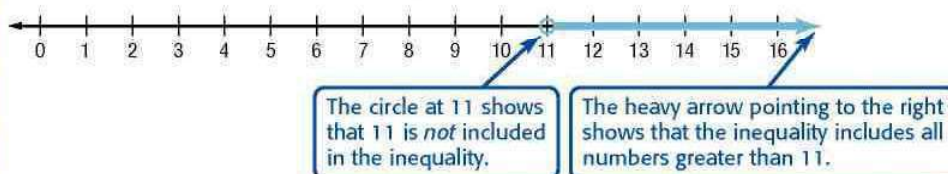
Solve $7 < x - 4$. Then graph it on a number line.

$$7 < x - 4 \quad \text{Original inequality}$$

$$7 + 4 < x - 4 + 4 \quad \text{Add 4 to each side.}$$

$$11 < x \quad \text{Simplify.}$$

Since $11 < x$ is the same as $x > 11$, the solution set is $\{x | x > 11\}$.



SOLVE INEQUALITIES BY SUBTRACTION Subtraction can also be used to solve inequalities.

Key Concept

Subtraction Property of Inequalities

- **Words** If any number is subtracted from each side of a true inequality, the resulting inequality is also true.
- **Symbols** For all numbers a , b , and c , the following are true.
 1. If $a > b$, then $a - c > b - c$.
 2. If $a < b$, then $a - c < b - c$.
- **Example**
$$\begin{array}{rcl} 17 & > & 8 \\ 17 - 5 & > & 8 - 5 \\ 12 & > & 3 \end{array}$$

This property is also true when $>$ and $<$ are replaced with $\geq$ and $\leq$.

Example 3 Solve by Subtracting

Solve $19 + r \geq 16$. Then graph the solution.

$$19 + r \geq 16 \quad \text{Original inequality}$$

$$19 + r - 19 \geq 16 - 19 \quad \text{Subtract 19 from each side.}$$

$$r \geq -3 \quad \text{Simplify.}$$

The solution set is $\{r | r \geq -3\}$.



Terms with variables can also be subtracted from each side to solve inequalities.

Example 4 Variables on Both Sides

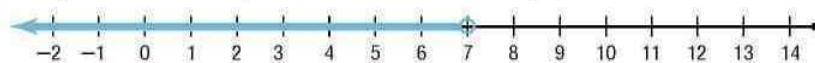
Solve $5p + 7 > 6p$. Then graph the solution.

$$5p + 7 > 6p \quad \text{Original inequality}$$

$$5p + 7 - 5p > 6p - 5p \quad \text{Subtract } 5p \text{ from each side.}$$

$$7 > p \quad \text{Simplify.}$$

Since $7 > p$ is the same as $p < 7$, the solution set is $\{p \mid p < 7\}$.



Verbal problems containing phrases like *greater than* or *less than* can often be solved by using inequalities. The following chart shows some other phrases that indicate inequalities.

Inequalities			
$<$	$>$	$\leq$	$\geq$
- less than - fewer than	- greater than - more than	- at most - no more than - less than or equal to	- at least - no less than - greater than or equal to

Example 5 Write and Solve an Inequality

Write an inequality for the sentence below. Then solve the inequality.

Four times a number is no more than three times that number plus eight.

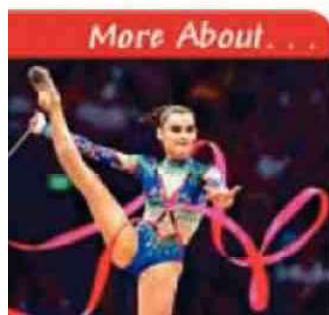
$$\underbrace{\text{Four times a number}}_{4n} \quad \underbrace{\text{is no more than}}_{\leq} \quad \underbrace{\text{three times that number}}_{3n} \quad \underbrace{\text{plus}}_{+} \quad \underbrace{\text{eight}}_{8}$$

$$4n \leq 3n + 8 \quad \text{Original inequality}$$

$$4n - 3n \leq 3n + 8 - 3n \quad \text{Subtract } 3n \text{ from each side.}$$

$$n \leq 8 \quad \text{Simplify.}$$

The solution set is $\{n \mid n \leq 8\}$.



Olympics

Yulia Barsukova of the Russian Federation won the gold medal in rhythmic gymnastics at the 2000 Summer Olympics in Sydney, and Yulia Raskina of Belarus won the silver medal.

Source: www.olympic.org

Example 6 Write an Inequality to Solve a Problem

OLYMPICS Yulia Raskina scored a total of 39.548 points in the four events of rhythmic gymnastics. Yulia Barsukova scored 9.883 in the rope competition, 9.900 in the hoop competition, and 9.916 in the ball competition. How many points did Barsukova need to score in the ribbon competition to surpass Raskina and win the gold medal?

Words Barsukova's total must be greater than Raskina's total.

Variable Let r = Barsukova's score in the ribbon competition.

$$\text{Inequality} \quad \underbrace{9.883 + 9.900 + 9.916 + r}_{\text{Barsukova's total}} \quad \underbrace{>}_{\text{is greater than}} \quad \underbrace{39.548}_{\text{Raskina's total.}}$$

Solve the inequality.

$$9.883 + 9.900 + 9.916 + r > 39.548$$

$$29.699 + r > 39.548$$

$$29.699 + r - 29.699 > 39.548 - 29.699$$

$$r > 9.849$$

Original inequality

Simplify.

Subtract 29.699 from each side.

Simplify.

Barsukova needed to score more than 9.849 points to win the gold medal.

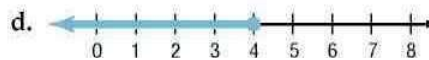
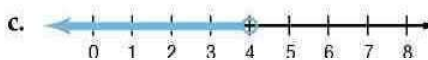
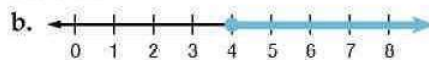
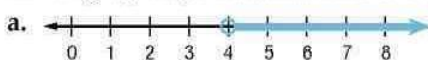
Check for Understanding

Concept Check

- OPEN ENDED** List three inequalities that are equivalent to $y < -3$.
- Compare and contrast the graphs of $a < 4$ and $a \leq 4$.
- Explain what $\{b \mid b \geq -5\}$ means.

Guided Practice

- Which graph represents the solution of $m + 3 > 7$?



Solve each inequality. Then check your solution, and graph it on a number line.

5. $a + 4 < 2$

6. $9 \leq b + 4$

7. $t - 7 \geq 5$

8. $y - 2.5 > 3.1$

9. $5.2r + 6.7 \geq 6.2r$

10. $7p \leq 6p - 2$

Define a variable, write an inequality, and solve each problem. Then check your solution.

- A number decreased by 8 is at most 14.
- A number plus 7 is greater than 2.

Application

- HEALTH** Chapa's doctor recommended that she limit her fat intake to no more than 60 grams per day. This morning, she ate two breakfast bars with 3 grams of fat each. For lunch she ate pizza with 21 grams of fat. If she follows her doctor's advice, how many grams of fat can she have during the rest of the day?

Practice and Apply

Homework Help

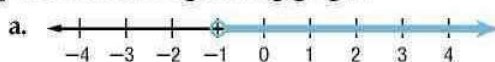
For Exercises	See Examples
14–39	1–4
40–45	5
46–55	6

Extra Practice

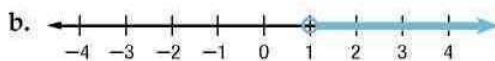
See page 833.

Match each inequality with its corresponding graph.

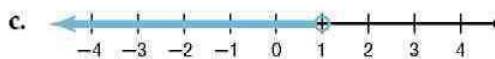
14. $x - 3 \geq -2$



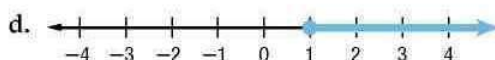
15. $x + 7 \leq 6$



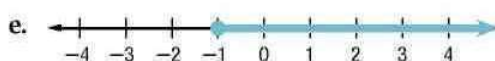
16. $4x > 3x - 1$



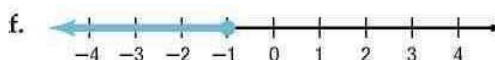
17. $8 + x < 9$



18. $5 \leq x + 6$



19. $x - 1 > 0$



Solve each inequality. Then check your solution, and graph it on a number line.

20. $t + 14 \geq 18$
 21. $d + 5 \leq 7$
 22. $n - 7 < -3$
 23. $s - 5 > -1$
 24. $5 < 3 + g$
 25. $4 > 8 + r$
 26. $-3 \geq q - 7$
 27. $2 \leq m - 1$
 28. $2y > -8 + y$
 29. $3f < -3 + 2f$
 30. $3b \leq 2b - 5$
 31. $4w \geq 3w + 1$
 32. $v - (-4) > 3$
 33. $a - (-2) \leq -3$
 34. $-0.23 < h - (-0.13)$
 35. $x + 1.7 \geq 2.3$
 36. $a + \frac{1}{4} > \frac{1}{8}$
 37. $p - \frac{2}{3} \leq \frac{4}{9}$
38. If $d + 5 \geq 17$, then complete each inequality.
a. $d \geq \underline{\quad ? \quad}$ b. $d + \underline{\quad ? \quad} \geq 20$ c. $d - 5 \geq \underline{\quad ? \quad}$
39. If $z - 2 \leq 10$, then complete each inequality.
a. $z \leq \underline{\quad ? \quad}$ b. $z - \underline{\quad ? \quad} \leq 5$ c. $z + 4 \leq \underline{\quad ? \quad}$

Define a variable, write an inequality, and solve each problem. Then check your solution.

40. The sum of a number and 13 is at least 27.
41. A number decreased by 5 is less than 33.
42. Thirty is no greater than the sum of a number and -8 .
43. Twice a number is more than the sum of that number and 14.
44. The sum of two numbers is at most 18, and one of the numbers is -7 .
45. Four times a number is less than or equal to the sum of three times the number and -2 .
46. **BIOLOGY** Adult Nile crocodiles weigh up to 2200 pounds. If a young Nile crocodile weighs 157 pounds, how many pounds might it be expected to gain in its lifetime?
47. **ASTRONOMY** There are at least 200 billion stars in the Milky Way. If 1100 of these stars can be seen in a rural area without the aid of a telescope, how many stars in the galaxy cannot be seen in this way?



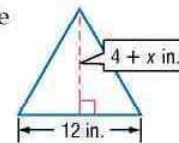
More About...

Biology

One common species of bees is the honeybee. A honeybee colony may have 60,000 to 80,000 bees.

Source: Penn State, Cooperative Extension Service

48. **BIOLOGY** There are 3500 species of bees and more than 600,000 species of insects. How many species of insects are not bees?
49. **BANKING** City Bank requires a minimum balance of \$1500 to maintain free checking services. If Mr. Hayashi knows he must write checks for \$1300 and \$947, how much money should he have in his account before writing the checks?
50. **GEOMETRY** The length of the base of the triangle at the right is less than the height of the triangle. What are the possible values of x ?



51. **SHOPPING** Terrell has \$65 to spend at the mall. He bought a T-shirt for \$18 and a belt for \$14. If Terrell still wants to buy a pair of jeans, how much can he spend on the jeans?
52. **SOCCER** The Centerville High School soccer team plays 18 games in the season. The team has a goal of winning at least 60% of its games. After the first three weeks of the season, the team has won 4 games. How many more games must the team win to meet their goal?

53. **CRITICAL THINKING** Determine whether each statement is *always*, *sometimes*, or *never* true.
- If $a < b$ and $c < d$, then $a + c < b + d$.
 - If $a < b$ and $c < d$, then $a + c \geq b + d$.
 - If $a < b$ and $c < d$, then $a - c = b - d$.

HEALTH For Exercises 54 and 55, use the following information.

Hector's doctor told him that his cholesterol level should be below 200. Hector's cholesterol is 225.

54. Let p represent the number of points Hector should lower his cholesterol. Write an inequality with $225 - p$ on one side.
55. Solve the inequality.
56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are inequalities used to describe school sports?

Include the following in your answer:

- an inequality describing the number of schools needed to add girls' track and field so that the number is greater than the number of schools currently participating in girls' basketball.



57. Which inequality is *not* equivalent to $x \leq 12$?
- (A) $x - 7 \leq 5$ (B) $x + 4 \leq 16$ (C) $x - 1 \leq 13$ (D) $12 \geq x$
58. Which statement is modeled by $n + 6 \geq 5$?
- (A) The sum of a number and six is at least five.
 (B) The sum of a number and six is at most five.
 (C) The sum of a number and six is greater than five.
 (D) The sum of a number and six is no greater than five.

Maintain Your Skills

- Mixed Review** 59. Would a scatter plot for the relationship of a person's height to the person's grade on the last math test show a *positive*, *negative*, or *no correlation*? (Lesson 5-7)

Write an equation in slope-intercept form of the line that passes through the given point and is parallel to the graph of each equation. (Lesson 5-6)

60. $(1, -3)$; $y = 3x - 2$ 61. $(0, 4)$; $x + y = -3$ 62. $(-1, 2)$; $2x - y = 1$

Find the next two terms in each sequence. (Lesson 4-8)

63. 7, 13, 19, 25, ... 64. 243, 81, 27, 9, ... 65. 3, 6, 12, 24, ...

Solve each equation if the domain is $\{-1, 3, 5\}$. (Lesson 4-4)

66. $y = -2x$ 67. $y = 7 - x$ 68. $2x - y = 6$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Solve each equation.

(For review of **multiplication and division equations**, see Lesson 3-3.)

69. $6g = 42$ 70. $\frac{t}{9} = 14$ 71. $\frac{2}{3}y = 14$ 72. $3m = 435$
 73. $\frac{4}{7}x = 28$ 74. $5.3g = 11.13$ 75. $\frac{a}{3.5} = 7$ 76. $8p = 35$





Algebra Activity

A Preview of Lesson 6-2

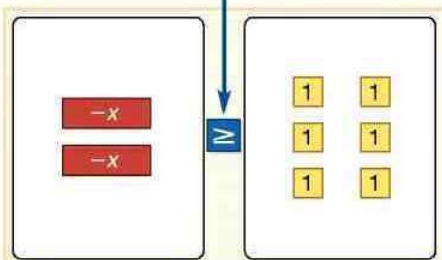
Solving Inequalities

You can use algebra tiles to solve inequalities.

Solve $-2x \geq 6$.

Step 1 Model the inequality.

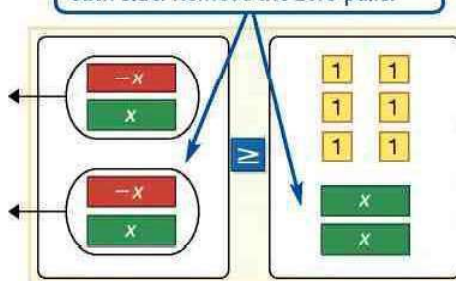
Use a self-adhesive note to cover the equals sign on the equation mat. Then write a $\geq$ symbol on the note. Model the inequality.



$$-2x \geq 6$$

Step 2 Remove zero pairs.

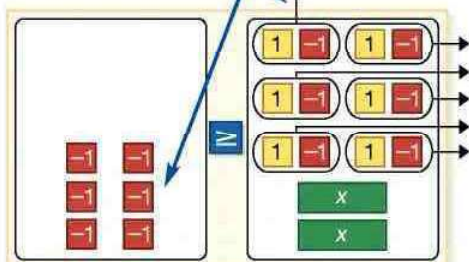
Since you do not want to solve for a negative x tile, eliminate the negative x tiles by adding 2 positive x tiles to each side. Remove the zero pairs.



$$-2x + 2x \geq 6 + 2x$$

Step 3 Remove zero pairs.

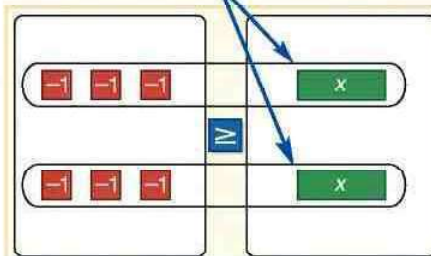
Add 6 negative 1 tiles to each side to isolate the x tiles. Remove the zero pairs.



$$-6 \geq 2x$$

Step 4 Group the tiles.

Separate the tiles into 2 groups.



$$-3 \geq x \text{ or } x \leq -3$$

Model and Analyze

Use algebra tiles to solve each inequality.

- $-4x < 12$
- $-2x > 8$
- $-3x \geq -6$
- $-5x \leq -5$
- In Exercises 1-4, is the coefficient of x in each inequality positive or negative?
- Compare the inequality symbols and locations of the variable in Exercises 1-4 with those in their solutions. What do you find?
- Model the solution for $2x \geq 6$. What do you find? How is this different from solving $-2x \geq 6$?

6-2

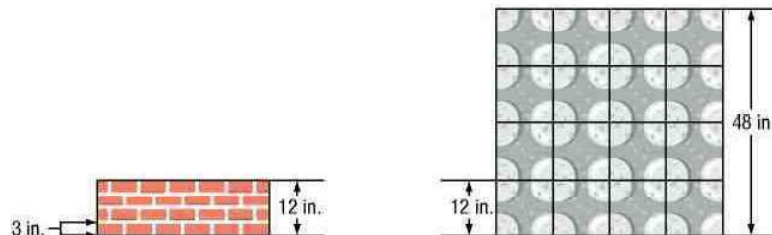
Solving Inequalities by Multiplication and Division

What You'll Learn

- Solve linear inequalities by using multiplication.
- Solve linear inequalities by using division.

Why are inequalities important in landscaping?

Isabel Franco is a landscape architect. To beautify a garden, she plans to build a decorative wall of either bricks or blocks. Each brick is 3 inches high, and each block is 12 inches high. Notice that $3 < 12$.



A wall 4 bricks high would be lower than a wall 4 blocks high.

$$3 \times 4 \quad ? \quad 12 \times 4$$

$$12 < 48$$

SOLVE INEQUALITIES BY MULTIPLICATION If each side of an inequality is multiplied by a positive number, the inequality remains true.

$$\begin{array}{rcl} 8 > 5 & & 5 < 9 \\ 8(2) \quad ? \quad 5(2) & \text{Multiply each side by 2.} & 5(4) \quad ? \quad 9(4) \quad \text{Multiply each side by 4.} \\ 16 > 10 & & 20 < 36 \end{array}$$

This is *not* true when multiplying by negative numbers.

$$\begin{array}{rcl} 5 > 3 & & -6 < 8 \\ 5(-2) \quad ? \quad 3(-2) & \text{Multiply each side by } -2. & -6(-5) \quad ? \quad 8(-5) \quad \text{Multiply each side by } -5. \\ -10 < -6 & & 30 > -40 \end{array}$$

If each side of an inequality is multiplied by a negative number, the direction of the inequality symbol changes. These examples illustrate the **Multiplication Property of Inequalities**.

Key Concept

Multiplying by a Positive Number

- Words** If each side of a true inequality is multiplied by the same positive number, the resulting inequality is also true.
- Symbols** If a and b are any numbers and c is a positive number, the following are true.
If $a > b$, then $ac > bc$, and if $a < b$, then $ac < bc$.

Key Concept**Multiplying by a Negative Number**

- **Words** If each side of a true inequality is multiplied by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is also true.
- **Symbols** If a and b are any numbers and c is a negative number, the following are true.
If $a > b$, then $ac < bc$, and if $a < b$, then $ac > bc$.

This property also holds for inequalities involving $\geq$ and $\leq$.

You can use this property to solve inequalities.

Example 1 Multiply by a Positive Number

Solve $\frac{b}{7} \geq 25$. Then check your solution.

$$\frac{b}{7} \geq 25 \quad \text{Original inequality}$$

$$(7)\frac{b}{7} \geq (7)25 \quad \text{Multiply each side by 7. Since we multiplied by a positive number, the inequality symbol stays the same.}$$

$$b \geq 175$$

CHECK To check this solution, substitute 175, a number less than 175, and a number greater than 175 into the inequality.

Let $b = 175$.

$$\frac{175}{7} \stackrel{?}{\geq} 25$$

$$25 \geq 25 \quad \checkmark$$

Let $b = 140$.

$$\frac{140}{7} \stackrel{?}{\geq} 25$$

$$20 \not\geq 25$$

Let $b = 210$.

$$\frac{210}{7} \stackrel{?}{\geq} 25$$

$$30 \geq 25 \quad \checkmark$$

The solution set is $\{b \mid b \geq 175\}$.

Study Tip**Common****Misconception**

A negative sign in an inequality does not necessarily mean that the direction of the inequality should change. For example, when solving $\frac{x}{6} > -3$, do *not* change the direction of the inequality.

Example 2 Multiply by a Negative Number

Solve $-\frac{2}{5}p < -14$.

$$-\frac{2}{5}p < -14 \quad \text{Original inequality}$$

$$\left(-\frac{5}{2}\right)\left(-\frac{2}{5}p\right) > \left(-\frac{5}{2}\right)(-14) \quad \text{Multiply each side by } -\frac{5}{2} \text{ and change } < \text{ to } >.$$

$$p > 35$$

The solution set is $\{p \mid p > 35\}$.

Example 3 Write and Solve an Inequality

Write an inequality for the sentence below. Then solve the inequality.

One fourth of a number is less than -7 .

$$\underbrace{\text{One fourth}}_{\frac{1}{4}} \underbrace{\text{of}}_{\times} \underbrace{\text{a number}}_n \underbrace{\text{is less than}}_{<} \underbrace{-7}_{-7}$$

$$\frac{1}{4}n < -7$$

Original inequality

$$(4)\frac{1}{4}n < (4)(-7) \quad \text{Multiply each side by 4 and do not change the inequality's direction.}$$

$$n < -28$$

The solution set is $\{n \mid n < -28\}$.

SOLVE INEQUALITIES BY DIVISION Dividing each side of an inequality by the same number is similar to multiplying each side of an equality by the same number. Consider the inequality $6 < 15$.

Divide each side by 3.

$$\begin{array}{rcl} 6 & < & 15 \\ 6 \div 3 & ? & 15 \div 3 \\ 2 & < & 5 \end{array}$$

Since each side is divided by a positive number, the direction of the inequality symbol remains the same.

Divide each side by -3 .

$$\begin{array}{rcl} 6 & < & 15 \\ 6 \div (-3) & ? & 15 \div (-3) \\ -2 & > & -5 \end{array}$$

Since each side is divided by a negative number, the direction of the inequality symbol is reversed.

These examples illustrate the **Division Property of Inequalities**.

Key Concept

Dividing by a Positive Number

- **Words** If each side of a true inequality is divided by the same positive number, the resulting inequality is also true.
- **Symbols** If a and b are any numbers and c is a positive number, the following are true.
If $a > b$, then $\frac{a}{c} > \frac{b}{c}$, and if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.

Dividing by a Negative Number

- **Words** If each side of a true inequality is divided by the same negative number, the direction of the inequality symbol must be reversed so that the resulting inequality is also true.
- **Symbols** If a and b are any numbers and c is a negative number, the following are true.
If $a > b$, then $\frac{a}{c} < \frac{b}{c}$, and if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

This property also holds for inequalities involving $\geq$ and $\leq$.

Example 4 Divide by a Positive Number

Solve $14h > 91$.

$$14h > 91 \quad \text{Original inequality}$$

$$\frac{14h}{14} > \frac{91}{14} \quad \text{Divide each side by 14 and do not change the direction of the inequality sign.}$$

$$h > 6.5$$

CHECK Let $h = 6.5$.

$$14h > 91$$

$$14(6.5) \stackrel{?}{>} 91$$

$$91 \nless 91$$

Let $h = 7$.

$$14h > 91$$

$$14(7) \stackrel{?}{>} 91$$

$$98 > 91 \quad \checkmark$$

Let $h = 6$.

$$14h > 91$$

$$14(6) \stackrel{?}{>} 91$$

$$84 \nless 91$$

The solution set is $\{h \mid h > 6.5\}$.

Since dividing is the same as multiplying by the reciprocal, there are two methods to solve an inequality that involve multiplication.

Example 5 Divide by a Negative NumberSolve $-5t \geq 275$ using two methods.**Method 1** Divide.

$$-5t \geq 275 \quad \text{Original inequality}$$

$$\frac{-5t}{-5} \leq \frac{275}{-5} \quad \text{Divide each side by } -5 \text{ and change } \geq \text{ to } \leq.$$

$$t \leq -55 \quad \text{Simplify.}$$

Method 2 Multiply by the multiplicative inverse.

$$-5t \geq 275 \quad \text{Original inequality}$$

$$\left(-\frac{1}{5}\right)(-5t) \leq \left(-\frac{1}{5}\right)275 \quad \text{Multiply each side by } -\frac{1}{5} \text{ and change } \geq \text{ to } \leq.$$

$$t \leq -55 \quad \text{Simplify.}$$

The solution set is $\{t \mid t \leq -55\}$.

You can use the Multiplication Property and the Division Property for Inequalities to solve standardized test questions.

Standardized Test Practice**A B C D****Example 6** The Word "not"**Multiple-Choice Test Item**Which inequality does *not* have the solution $\{y \mid y \leq -5\}$?

- (A)** $-7y \geq 35$
 (B) $2y \leq -10$
 (C) $\frac{7}{5}y \geq -7$
 (D) $-\frac{y}{4} \geq \frac{5}{4}$

Read the Test ItemYou want to find the inequality that does *not* have the solution set $\{y \mid y \leq -5\}$.**Solve the Test Item**

Consider each possible choice.

$$\begin{aligned} \text{(A)} \quad -7y &\geq 35 \\ \frac{-7y}{-7} &\leq \frac{35}{-7} \\ y &\leq -5 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{(B)} \quad 2y &\leq -10 \\ \frac{2y}{2} &\leq \frac{-10}{2} \\ y &\leq -5 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{(C)} \quad \frac{7}{5}y &\geq -7 \\ \left(\frac{5}{7}\right)\left(\frac{7}{5}y\right) &\geq \left(\frac{5}{7}\right)(-7) \\ y &\geq -5 \end{aligned}$$

$$\begin{aligned} \text{(D)} \quad -\frac{y}{4} &\geq \frac{5}{4} \\ (-4)\left(-\frac{y}{4}\right) &\leq (-4)\left(\frac{5}{4}\right) \\ y &\leq -5 \quad \checkmark \end{aligned}$$

The answer is C.

The Princeton Review**Test-Taking Tip**

Always look for the word *not* in the questions. This indicates that you are looking for the one incorrect answer, rather than looking for the one correct answer. The word *not* is usually in italics or uppercase letters to draw your attention to it.

Check for Understanding**Concept Check**

1. **Explain** why you can use either the Multiplication Property of Inequalities or the Division Property of Inequalities to solve $-7r \leq 28$.

2. **OPEN ENDED** Write a problem that can be represented by the inequality $\frac{3}{4}c > 9$.

3. **FIND THE ERROR** Ilonia and Zachary are solving $-9b \leq 18$.

Ilonia	Zachary
$-9b \leq 18$	$-9b \leq 18$
$\frac{-9b}{-9} \geq \frac{18}{-9}$	$\frac{-9b}{-9} \leq \frac{18}{-9}$
$b \geq -2$	$b \leq -2$

Who is correct? Explain your reasoning.

Guided Practice

4. Which statement is represented by $7n \geq 14$?
- Seven times a number is at least 14.
 - Seven times a number is greater than 14.
 - Seven times a number is at most 14.
 - Seven times a number is less than 14.
5. Which inequality represents *five times a number is less than 25*?
- $5n > 25$
 - $5n \geq 25$
 - $5n < 25$
 - $5n \leq 25$

Solve each inequality. Then check your solution.

6. $-15g > 75$ 7. $\frac{t}{9} < -12$ 8. $-\frac{2}{3}b \leq -9$ 9. $25f \geq 9$

Define a variable, write an inequality, and solve each problem. Then check your solution.

10. The opposite of four times a number is more than 12.
11. Half of a number is at least 26.

Standardized Test Practice

12. Which inequality does *not* have the solution $\{x | x > 4\}$?
- $-5x < -20$
 - $6x < 24$
 - $\frac{1}{5}x > \frac{4}{5}$
 - $-\frac{3}{4}x < -3$

Practice and Apply

Homework Help

For Exercises	See Examples
13–18, 39–44	3
19–38 45–51	1, 2, 4, 5 6

Extra Practice

See page 833.

Match each inequality with its corresponding statement.

- | | |
|----------------------------|--|
| 13. $\frac{1}{5}n > 10$ | a. Five times a number is less than or equal to ten. |
| 14. $5n \leq 10$ | b. One fifth of a number is no less than ten. |
| 15. $5n > 10$ | c. Five times a number is less than ten. |
| 16. $-5n < 10$ | d. One fifth of a number is greater than ten. |
| 17. $\frac{1}{5}n \geq 10$ | e. Five times a number is greater than ten. |
| 18. $5n < 10$ | f. Negative five times a number is less than ten. |

Solve each inequality. Then check your solution.

- | | | | |
|-----------------------------|---------------------------|-------------------------------------|------------------------------------|
| 19. $6g \leq 144$ | 20. $7t > 84$ | 21. $-14d \geq 84$ | 22. $-16z \leq -64$ |
| 23. $\frac{m}{5} \geq 7$ | 24. $\frac{b}{10} \leq 5$ | 25. $-\frac{r}{7} < -7$ | 26. $-\frac{a}{11} > 9$ |
| 27. $\frac{5}{8}y \geq -15$ | 28. $\frac{2}{3}v < 6$ | 29. $-\frac{3}{4}q \leq -33$ | 30. $-\frac{2}{5}p > 10$ |
| 31. $-2.5w < 6.8$ | 32. $-0.8s > 6.4$ | 33. $\frac{15c}{-7} > \frac{3}{14}$ | 34. $\frac{4m}{5} < \frac{-3}{15}$ |

35. Solve $-\frac{y}{8} > \frac{1}{2}$. Then graph the solution.
36. Solve $-\frac{m}{9} \leq -\frac{1}{3}$. Then graph the solution.
37. If $2a \geq 7$, then complete each inequality.
 a. $a \geq \underline{\quad ? \quad}$ b. $-4a \leq \underline{\quad ? \quad}$ c. $\underline{\quad ? \quad} a \leq -21$
38. If $4t < -2$, then complete each inequality.
 a. $t < \underline{\quad ? \quad}$ b. $-8t > \underline{\quad ? \quad}$ c. $\underline{\quad ? \quad} t > 14$

Define a variable, write an inequality, and solve each problem. Then check your solution.

39. Seven times a number is greater than 28.
40. Negative seven times a number is at least 14.
41. Twenty-four is at most a third of a number.
42. Two thirds of a number is less than -15 .
43. Twenty-five percent of a number is greater than or equal to 90.
44. Forty percent of a number is less than or equal to 45.
45. **GEOMETRY** The area of a rectangle is less than 85 square feet. The length of the rectangle is 20 feet. What is the width of the rectangle?
46. **FUND-RAISING** The Middletown Marching Mustangs want to make at least \$2000 on their annual mulch sale. The band makes \$2.50 on each bag of mulch that is sold. How many bags of mulch should the band sell?
47. **LONG-DISTANCE COSTS** Juan's long-distance phone company charges him 9¢ for each minute or any part of a minute. He wants to call his friend, but he does not want to spend more than \$2.50 on the call. How long can he talk to his friend?
48. **EVENT PLANNING** The Country Corner Reception Hall does not charge a rental fee as long as at least \$4000 is spent on food. Shaniqua is planning a class reunion. If she has chosen a buffet that costs \$28.95 per person, how many people must attend the reunion to avoid a rental fee for the hall?
49. **LANDSCAPING** Matthew is planning a circular flower garden with a low fence around the border. If he can use up to 38 feet of fence, what radius can he use for the garden? (*Hint: $C = 2\pi r$*)
50. **DRIVING** Average speed is calculated by dividing distance by time. If the speed limit on the interstate is 65 miles per hour, how far can a person travel legally in $1\frac{1}{2}$ hours?
51. **ZOOS** The yearly membership to the San Diego Zoo for a family with 2 adults and 2 children is \$144. The regular admission to the zoo is \$18 for each adult and \$8 for each child. How many times should such a family plan to visit the zoo in a year to make a membership less expensive than paying regular admission?
52. **CRITICAL THINKING** Give a counterexample to show that each statement is not always true.
 a. If $a > b$, then $a^2 > b^2$. b. If $a < b$ and $c < d$, then $ac < bd$.
53. **CITY PLANNING** The city of Santa Clarita requires that a parking lot can have no more than 20% of the parking spaces limited to compact cars. If a certain parking lot has 35 spaces for compact cars, how many spaces must the lot have to conform to the code?

More About...



Zoos

Dr. Harry Wegeforth founded the San Diego Zoo in 1916 with just 50 animals. Today, the zoo has over 3800 animals.

Source: www.sandiegozoo.org

54. **CIVICS** For a candidate to run for a county office, he or she must submit a petition with at least 6000 signatures of registered voters. Usually only 85% of the signatures are valid. How many signatures should a candidate seek on a petition?

55. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

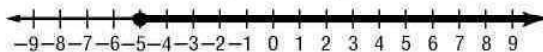
Why are inequalities important in landscaping?

Include the following in your answer:

- an inequality representing a brick wall that can be no higher than 4 feet, and
- an explanation of how to solve the inequality.



56. The solution set for which inequality is *not* represented by the following graph?



- (A) $-\frac{x}{5} \leq 1$ (B) $\frac{x}{5} \leq -1$ (C) $-9x \leq 45$ (D) $2.5x \geq -12.5$
57. Solve $-\frac{7}{8}t < \frac{14}{15}$.
- (A) $\left\{t \mid t > \frac{16}{15}\right\}$ (B) $\left\{t \mid t < \frac{16}{15}\right\}$ (C) $\left\{t \mid t > -\frac{16}{15}\right\}$ (D) $\left\{t \mid t < -\frac{16}{15}\right\}$

Maintain Your Skills

Mixed Review

Solve each inequality. Then check your solution, and graph it on a number line.

(Lesson 6-1)

58. $s - 7 < 12$

59. $g + 3 \leq -4$

60. $7 > n + 2$

61. Draw a scatter plot that shows a positive correlation. (Lesson 5-7)

Write an equation in standard form for a line that passes through each pair of points. (Lesson 5-4)

62. $(-1, 3), (2, 4)$

63. $(5, -2), (-1, -2)$

64. $(3, 3), (-1, 2)$

If $h(x) = 3x + 2$, find each value. (Lesson 4-6)

65. $h(-4)$

66. $h(2)$

67. $h(w)$

68. $h(r - 6)$

Solve each proportion. (Lesson 3-6)

69. $\frac{3}{4} = \frac{x}{8}$

70. $\frac{t}{1.5} = \frac{2.4}{1.6}$

71. $\frac{w + 2}{5} = \frac{7}{5}$

72. $\frac{x}{3} = \frac{x + 5}{15}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Solve each equation.

(To review *multi-step equations*, see Lessons 3-4 and 3-5.)

73. $5x - 3 = 32$

74. $4t + 9 = 14$

75. $6y - 1 = 4y + 23$

76. $\frac{14g + 5}{6} = 9$

77. $5a + 6 = 9a - (7a + 18)$

78. $2(p - 4) = 7(p + 3)$

Practice Quiz 1

Lessons 6-1 and 6-2

Solve each inequality. Then check your solution, and graph it on a number line. (Lesson 6-1)

1. $h - 16 > -13$

2. $r + 3 \leq -1$

3. $4 \geq p + 9$

4. $-3 < a - 5$

5. $7g \leq 6g - 1$

Solve each inequality. Then check your solution. (Lesson 6-2)

6. $15z \geq 105$

7. $\frac{n}{5} < 7$

8. $-\frac{3}{7}q > 15$

9. $-156 < 12r$

10. $-\frac{2}{5}w \leq -\frac{1}{2}$

6-3

Solving Multi-Step Inequalities

What You'll Learn

- Solve linear inequalities involving more than one operation.
- Solve linear inequalities involving the Distributive Property.

How are linear inequalities used in science?

The boiling point of a substance is the temperature at which the element changes from a liquid to a gas. The boiling point of chlorine is -31°F . That means chlorine will be a gas for all temperatures greater than -31°F . If F represents temperature in degrees Fahrenheit, the inequality $F > -31$ represents the temperatures for which chlorine is a gas.

If C represents degrees Celsius, then $F = \frac{9}{5}C + 32$. You can solve $\frac{9}{5}C + 32 > -31$ to find the temperatures in degrees Celsius for which chlorine is a gas.



argon	-303°F
chlorine	-31°F
bromine	138°F
water	212°F
iodine	363°F

Source: World Book Encyclopedia

SOLVE MULTI-STEP INEQUALITIES The inequality $\frac{9}{5}C + 32 > -31$ involves more than one operation. It can be solved by undoing the operations in the same way you would solve an equation with more than one operation.

Example 1 Solve a Real-World Problem

SCIENCE Find the temperatures in degrees Celsius for which chlorine is a gas.

$$\frac{9}{5}C + 32 > -31 \quad \text{Original inequality}$$

$$\frac{9}{5}C + 32 - 32 > -31 - 32 \quad \text{Subtract 32 from each side.}$$

$$\frac{9}{5}C > -63 \quad \text{Simplify.}$$

$$\left(\frac{5}{9}\right)\frac{9}{5}C > \left(\frac{5}{9}\right)(-63) \quad \text{Multiply each side by } \frac{5}{9}.$$

$$C > -35 \quad \text{Simplify.}$$

Chlorine will be a gas for all temperatures greater than -35°C .

When working with inequalities, do not forget to reverse the inequality sign whenever you multiply or divide each side by a negative number.

Example 2 Inequality Involving a Negative Coefficient

Solve $-7b + 19 < -16$. Then check your solution.

$$-7b + 19 < -16 \quad \text{Original inequality}$$

$$-7b + 19 - 19 < -16 - 19 \quad \text{Subtract 19 from each side.}$$

$$-7b < -35 \quad \text{Simplify.}$$

$$\frac{-7b}{-7} > \frac{-35}{-7} \quad \text{Divide each side by } -7 \text{ and change } < \text{ to } >.$$

$$b > 5 \quad \text{Simplify.}$$

CHECK To check this solution, substitute 5, a number less than 5, and a number greater than 5.

Let $b = 5$.

$$\begin{aligned} -7b + 19 &< -16 \\ -7(5) + 19 &\stackrel{?}{<} -16 \\ -35 + 19 &\stackrel{?}{<} -16 \\ -16 &\nless -16 \end{aligned}$$

Let $b = 4$.

$$\begin{aligned} -7b + 19 &< -16 \\ -7(4) + 19 &\stackrel{?}{<} -16 \\ -28 + 19 &\stackrel{?}{<} -16 \\ -9 &\nless -16 \end{aligned}$$

Let $b = 6$.

$$\begin{aligned} -7b + 19 &< -16 \\ -7(6) + 19 &\stackrel{?}{<} -16 \\ -42 + 19 &\stackrel{?}{<} -16 \\ -23 &< -16 \quad \checkmark \end{aligned}$$

The solution set is $\{b \mid b > 5\}$.

Example 3 Write and Solve an Inequality

Write an inequality for the sentence below. Then solve the inequality.
Three times a number minus eighteen is at least five times the number plus twenty-one.

$$\begin{array}{ccccccc} \text{Three times} & & \text{minus} & & \text{is at} & & \text{five times} \\ \text{a number} & & & & \text{least} & & \text{the number} \\ \hline 3n & - & 18 & \geq & 5n & + & 21 \end{array}$$

$$3n - 18 \geq 5n + 21 \quad \text{Original inequality}$$

$$3n - 18 - 5n \geq 5n + 21 - 5n \quad \text{Subtract } 5n \text{ from each side.}$$

$$-2n - 18 \geq 21 \quad \text{Simplify.}$$

$$-2n - 18 + 18 \geq 21 + 18 \quad \text{Add 18 to each side.}$$

$$-2n \geq 39 \quad \text{Simplify.}$$

$$\frac{-2n}{-2} \leq \frac{39}{-2} \quad \text{Divide each side by } -2 \text{ and change } \geq \text{ to } \leq.$$

$$n \leq -19.5 \quad \text{Simplify.}$$

The solution set is $\{n \mid n \leq -19.5\}$.

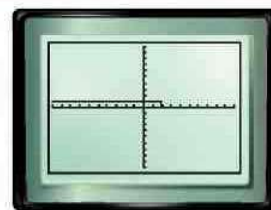
A graphing calculator can be used to solve inequalities.



Graphing Calculator Investigation

Solving Inequalities

You can find the solution of an inequality in one variable by using a graphing calculator. On a TI-83 Plus, clear the $Y =$ list. Enter $6x + 9 < -4x + 29$ as $Y1$. (The symbol $<$ is item 5 on the TEST menu.) Press **GRAPH**.



$[10, 10]$ scl: 1 by $[10, 10]$ scl: 1

Think and Discuss

1. Describe what is shown on the screen.
2. Use the TRACE function to scan the values along the graph. What do you notice about the values of y on the graph?
3. Solve the inequality algebraically. How does your solution compare to the pattern you noticed in Exercise 2?

SOLVE INEQUALITIES INVOLVING THE DISTRIBUTIVE PROPERTY

When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example 4 Distributive Property

Solve $3d - 2(8d - 9) > 3 - (2d + 7)$.

$$3d - 2(8d - 9) > 3 - (2d + 7) \quad \text{Original inequality}$$

$$3d - 16d + 18 > 3 - 2d - 7 \quad \text{Distributive Property}$$

$$-13d + 18 > -2d - 4 \quad \text{Combine like terms.}$$

$$-13d + 18 + 13d > -2d - 4 + 13d \quad \text{Add } 13d \text{ to each side.}$$

$$18 > 11d - 4 \quad \text{Simplify.}$$

$$18 + 4 > 11d - 4 + 4 \quad \text{Add 4 to each side.}$$

$$22 > 11d \quad \text{Simplify.}$$

$$\frac{22}{11} > \frac{11d}{11} \quad \text{Divide each side by 11.}$$

$$2 > d \quad \text{Simplify.}$$

Since $2 > d$ is the same as $d < 2$, the solution set is $\{d \mid d < 2\}$.

If solving an inequality results in a statement that is always true, the solution is all real numbers. If solving an inequality results in a statement that is never true, the solution is the empty set $\emptyset$. The empty set has no members.

Example 5 Empty Set

Solve $8(t + 2) - 3(t - 4) < 5(t - 7) + 8$.

$$8(t + 2) - 3(t - 4) < 5(t - 7) + 8 \quad \text{Original inequality}$$

$$8t + 16 - 3t + 12 < 5t - 35 + 8 \quad \text{Distributive Property}$$

$$5t + 28 < 5t - 27 \quad \text{Combine like terms.}$$

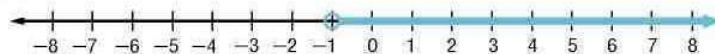
$$5t + 28 - 5t < 5t - 27 - 5t \quad \text{Subtract } 5t \text{ from each side.}$$

$$28 < -27 \quad \text{This statement is false.}$$

Since the inequality results in a false statement, the solution set is the empty set $\emptyset$.

Check for Understanding

- Concept Check**
1. Compare and contrast the method used to solve $-5h + 6 = -7$ and the method used to solve $-5h + 6 \leq -7$.
 2. **OPEN ENDED** Write a multi-step inequality with the solution graphed below.



- Guided Practice**
3. Justify each indicated step.

$$3(a - 7) + 9 \leq 21$$

$$3a - 21 + 9 \leq 21 \quad \text{a. } \underline{\hspace{1cm}}$$

$$3a - 12 \leq 21$$

$$3a - 12 + 12 \leq 21 + 12 \quad \text{b. } \underline{\hspace{1cm}}$$

$$3a \leq 33$$

$$\frac{3a}{3} \leq \frac{33}{3} \quad \text{c. } \underline{\hspace{1cm}}$$

$$a \leq 11$$

Solve each inequality. Then check your solution.

4. $-4y - 23 < 19$ 5. $\frac{2}{3}r + 9 \geq -3$ 6. $7b + 11 > 9b - 13$
 7. $-5(g + 4) > 3(g - 4)$ 8. $3 + 5t \leq 3(t + 1) - 4(2 - t)$

9. Define a variable, write an inequality, and solve the problem below. Then check your solution.

Seven minus two times a number is less than three times the number plus thirty-two.

- Application** 10. **SALES** A salesperson is paid \$22,000 a year plus 5% of the amount of sales made. What is the amount of sales needed to have an annual income greater than \$35,000?

Practice and Apply

Homework Help

For Exercises	See Examples
11–14	1–5
15–34	2, 4, 5
35–38	3
39–52	1

Extra Practice

See page 834.

Justify each indicated step.

11. $\frac{2}{5}w + 7 \leq -9$
 $\frac{2}{5}w + 7 - 7 \leq -9 - 7$ a. $\underline{\quad ? \quad}$
 $\frac{2}{5}w \leq -16$
 $\left(\frac{5}{2}\right)\frac{2}{5}w \leq \left(\frac{5}{2}\right)(-16)$ b. $\underline{\quad ? \quad}$
 $w \leq -40$

12. $m > \frac{15 - 2m}{-3}$
 $(-3)m < (-3)\frac{15 - 2m}{-3}$ a. $\underline{\quad ? \quad}$
 $-3m < 15 - 2m$
 $-3m + 2m < 15 - 2m + 2m$ b. $\underline{\quad ? \quad}$
 $-m < 15$
 $(-1)(-m) > (-1)15$ c. $\underline{\quad ? \quad}$
 $m > -15$

13. Solve $4(t - 7) \leq 2(t + 9)$. Show each step and justify your work.
 14. Solve $-5(k + 4) > 3(k - 4)$. Show each step and justify your work.

Solve each inequality. Then check your solution.

15. $-3t + 6 \leq -3$ 16. $-5 - 8f > 59$ 17. $-2 - \frac{d}{5} < 23$
 18. $\frac{w}{8} - 13 > -6$ 19. $7q - 1 + 2q \leq 29$ 20. $8a + 2 - 10a \leq 20$
 21. $9r + 15 \leq 24 + 10r$ 22. $13k - 11 > 7k + 37$ 23. $\frac{2v - 3}{5} \geq 7$
 24. $\frac{3a + 8}{2} < 10$ 25. $\frac{3w + 5}{4} \geq 2w$ 26. $\frac{5b + 8}{3} < 3b$
 27. $7 + 3t \leq 2(t + 3) - 2(-1 - t)$ 28. $5(2h - 6) - 7(h + 7) > 4h$
 29. $3y + 4 > 2(y + 3) + y$ 30. $3 - 3(b - 2) < 13 - 3(b - 6)$
 31. $3.1v - 1.4 \geq 1.3v + 6.7$ 32. $0.3(d - 2) - 0.8d > 4.4$

33. Solve $4(y + 1) - 3(y - 5) \geq 3(y - 1)$. Then graph the solution.
 34. Solve $5(x + 4) - 2(x + 6) \geq 5(x + 1) - 1$. Then graph the solution.

Define a variable, write an inequality, and solve each problem. Then check your solution.

35. One eighth of a number decreased by five is at least thirty.
 36. Two thirds of a number plus eight is greater than twelve.
 37. Negative four times a number plus nine is no more than the number minus twenty-one.
 38. Three times the sum of a number and seven is greater than five times the number less thirteen.

GEOMETRY For Exercises 39 and 40, use the following information.

By definition, the measure of any acute angle is less than 90 degrees. Suppose the measure of an acute angle is $3a - 15$.

39. Write an inequality to represent the situation.
40. Solve the inequality.

SCHOOL For Exercises 41 and 42, use the following information.

Carmen's scores on three math tests were 91, 95, and 88. The fourth and final test of the grading period is tomorrow. She needs an average (mean) of at least 92 to receive an A for the grading period.

41. If s is her score on the fourth test, write an inequality to represent the situation.
42. If Carmen wants an A in math, what must she score on the test?

PHYSICAL SCIENCE For Exercises 43 and 44, use the information at the left and the information below.

The melting point for an element is the temperature where the element changes from a solid to a liquid. If C represents degrees Celsius and F represents degrees Fahrenheit, then $C = \frac{5(F - 32)}{9}$.

43. Write an inequality that can be used to find the temperatures in degrees Fahrenheit for which mercury is a solid.
44. For what temperatures will mercury be a solid?
45. **HEALTH** Keith weighs 200 pounds. He wants to weigh less than 175 pounds. If he can lose an average of 2 pounds per week on a certain diet, how long should he stay on his diet to reach his goal weight?
46. **CRITICAL THINKING** Write a multi-step inequality that has no solution and one that has infinitely many solutions.
47. **PERSONAL FINANCES** Nicholas wants to order a pizza. He has a total of \$13.00 to pay the delivery person. The pizza costs \$7.50 plus \$1.25 per topping. If he plans to tip 15% of the total cost of the pizza, how many toppings can he order?

LABOR For Exercises 48–50, use the following information.

A union worker made \$500 per week. His union sought a one-year contract and went on strike. Once the new contract was approved, it provided for a 4% raise.

48. Assume that the worker was not paid during the strike. Given his raise in salary, how many weeks could he strike and still make at least as much for the next 52 weeks as he would have made without a strike?
49. How would your answer to Exercise 48 change if the worker had been making \$600 per week?
50. How would your answer to Exercise 48 change if the worker's union provided him with \$150 per week during the strike?
51. **NUMBER THEORY** Find all sets of two consecutive positive odd integers whose sum is no greater than 18.
52. **NUMBER THEORY** Find all sets of three consecutive positive even integers whose sum is less than 40.



Physical Science

Mercury is a metal that is a liquid at room temperature. In fact, its melting point is -38°C . Mercury is used in thermometers because it expands evenly as it is heated.

Source: World Book Encyclopedia

53. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are linear inequalities used in science?

Include the following in your answer:

- an inequality for the temperatures in degrees Celsius for which bromine is a gas, and
- a description of a situation in which a scientist might use an inequality.



54. What is the first step in solving $\frac{y-5}{9} \geq 13$?

- (A) Add 5 to each side. (B) Subtract 5 from each side.
(C) Divide each side by 9. (D) Multiply each side by 9.

55. Solve $4t + 2 < 8t - (6t - 10)$.

- (A) $\{t | t < -6\}$ (B) $\{t | t > -6\}$ (C) $\{t | t < 4\}$ (D) $\{t | t > 4\}$



Graphing Calculator

Use a graphing calculator to solve each inequality.

56. $3x + 7 > 4x + 9$ 57. $13x - 11 \leq 7x + 37$ 58. $2(x - 3) < 3(2x + 2)$

Maintain Your Skills

Mixed Review

59. **BUSINESS** The charge per mile for a compact rental car at Great Deal Rentals is \$0.12. Mrs. Ludlow must rent a car for a business trip. She has a budget of \$50 for mileage charges. How many miles can she travel without going over her budget? (Lesson 6-2)

Solve each inequality. Then check your solution, and graph it on a number line.

(Lesson 6-1)

60. $d + 13 \geq 22$ 61. $t - 5 < 3$ 62. $4 > y + 7$

Write the standard form of an equation of the line that passes through the given point and has the given slope. (Lesson 5-5)

63. $(1, -3), m = 2$ 64. $(-2, -1), m = -\frac{2}{3}$ 65. $(3, 6), m = 0$

Determine the slope of the line that passes through each pair of points. (Lesson 5-1)

66. $(3, -1), (4, -6)$ 67. $(-2, -4), (1, 3)$ 68. $(0, 3), (-2, -5)$

Determine whether each equation is a linear equation. If an equation is linear, rewrite it in the form $Ax + By = C$. (Lesson 4-5)

69. $4x = 7 + 2y$ 70. $2x^2 - y = 7$ 71. $x = 12$

Solve each equation. Then check your solution. (Lesson 3-5)

72. $2(x - 2) = 3x - (4x - 5)$ 73. $5t - 7 = t + 3$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Graph each set of numbers on a number line.

(To review **graphing integers on a number line**, see Lesson 2-1.)

74. $\{-2, 3, 5\}$ 75. $\{-1, 0, 3, 4\}$ 76. $\{-5, -4, -1, 1\}$
77. $\{\text{integers less than } 5\}$ 78. $\{\text{integers greater than } -2\}$
79. $\{\text{integers between } 1 \text{ and } 6\}$ 80. $\{\text{integers between } -4 \text{ and } 2\}$
81. $\{\text{integers greater than or equal to } -4\}$
82. $\{\text{integers less than } 6 \text{ but greater than } -1\}$





Reading Mathematics

Compound Statements

Two simple statements connected by the words *and* or *or* form a compound statement. Before you can determine whether a compound statement is true or false, you must understand what the words *and* and *or* mean. Consider the statement below.

A triangle has three sides, *and* a hexagon has five sides.

For a compound statement connected by the word *and* to be true, both simple statements must be true. In this case, it is true that a triangle has three sides. However, it is false that a hexagon has five sides; it has six. Thus, the compound statement is false.

A compound statement connected by the word *or* may be *exclusive* or *inclusive*. For example, the statement "With your dinner, you may have soup *or* salad," is exclusive. In everyday language, *or* means one or the other, but not both. However, in mathematics, *or* is inclusive. It means one or the other or both. Consider the statement below.

A triangle has three sides, *or* a hexagon has five sides.

For a compound statement connected by the word *or* to be true, at least one of the simple statements must be true. Since it is true that a triangle has three sides, the compound statement is true.



Triangle



Square



Pentagon



Hexagon



Octagon

Reading to Learn

Determine whether each compound statement is *true* or *false*. Explain your answer.

1. A hexagon has six sides, *or* an octagon has seven sides.
2. An octagon has eight sides, *and* a pentagon has six sides.
3. A pentagon has five sides, *and* a hexagon has six sides.
4. A triangle has four sides, *or* an octagon does *not* have seven sides.
5. A pentagon has three sides, *or* an octagon has ten sides.
6. A square has four sides, *or* a hexagon has six sides.
7. $5 < 4$ or $8 < 6$
8. $-1 > 0$ and $1 < 5$
9. $4 > 0$ and $-4 < 0$
10. $0 = 0$ or $-2 > -3$
11. $5 \neq 5$ or $-1 > -4$
12. $0 > 3$ and $2 > -2$

6-4

Solving Compound Inequalities

What You'll Learn

- Solve compound inequalities containing the word *and* and graph their solution sets.
- Solve compound inequalities containing the word *or* and graph their solution sets.

Vocabulary

- compound inequality
- intersection
- union

How are compound inequalities used in tax tables?

Richard Kelley is completing his income tax return. He uses the table to determine the amount he owes in federal income tax.

If taxable income is—		Single	Married filing jointly	Married filing separately	Head of a household
At least	Less than				
41,000	41,050	8140	6154	8689	6996
41,050	41,100	8154	6161	8703	7010
41,100	41,150	8168	6169	8717	7024
41,150	41,200	8182	6176	8731	7038
41,200	41,250	8196	6184	8754	7052
41,250	41,300	8210	6191	8759	7066
41,300	41,350	8224	6199	8773	7080
41,350	41,400	8238	6206	8787	7094
41,400	41,450	8252	6214	8801	7108
41,450	41,500	8266	6221	8815	7122
41,500	41,550	8280	6229	8829	7136
41,550	41,600	8294	6236	8843	7150

Source: IRS

Let c represent the amount of Mr. Kelley's income. His income is at least \$41,350 and it is less than \$41,400. This can be written as $c \geq 41,350$ and $c < 41,400$. When considered together, these two inequalities form a **compound inequality**. This compound inequality can be written without using *and* in two ways.

$$41,350 \leq c < 41,400 \text{ or } 41,400 > c \geq 41,350$$

Study Tip

Reading Math

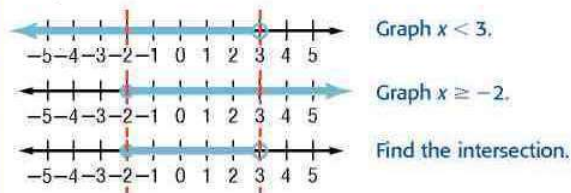
The statement $41,350 \leq c < 41,400$ can be read *41,350 is less than or equal to c , which is less than 41,400*.

INEQUALITIES CONTAINING AND A compound inequality containing *and* is true only if both inequalities are true. Thus, the graph of a compound inequality containing *and* is the **intersection** of the graphs of the two inequalities. In other words, the solution must be a solution of *both* inequalities.

The intersection can be found by graphing each inequality and then determining where the graphs overlap.

Example 1 Graph an Intersection

Graph the solution set of $x < 3$ and $x \geq -2$.



The solution set is $\{x \mid -2 \leq x < 3\}$. Note that the graph of $x \geq -2$ includes the point -2 . The graph of $x < 3$ does *not* include 3.

Study Tip

Reading Math

When solving problems involving inequalities,

- *within* is meant to be inclusive. Use $\leq$ or $\geq$.
- *between* is meant to be exclusive. Use $<$ or $>$.

Example 2 Solve and Graph an Intersection

Solve $-5 < x - 4 < 2$. Then graph the solution set.

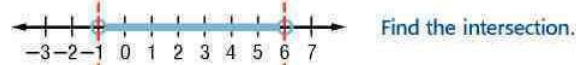
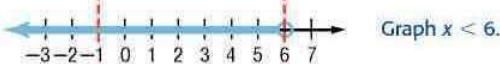
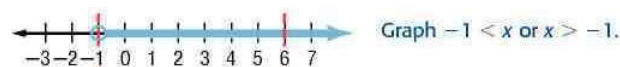
First express $-5 < x - 4 < 2$ using *and*. Then solve each inequality.

$$-5 < x - 4 \quad \text{and} \quad x - 4 < 2$$

$$-5 + 4 < x - 4 + 4 \quad x - 4 + 4 < 2 + 4$$

$$-1 < x \quad x < 6$$

The solution set is the intersection of the two graphs.



The solution set is $\{x \mid -1 < x < 6\}$.

INEQUALITIES CONTAINING OR Another type of compound inequality contains the word *or*. A compound inequality containing *or* is true if one or more of the inequalities is true. The graph of a compound inequality containing *or* is the **union** of the graphs of the two inequalities. In other words, the solution of the compound inequality is a solution of *either* inequality, not necessarily both.

The union can be found by graphing each inequality.

Example 3 Write and Graph a Compound Inequality

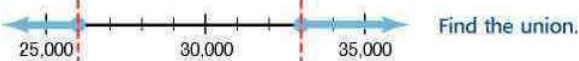
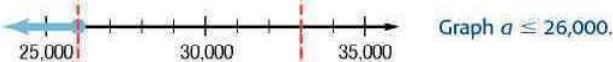
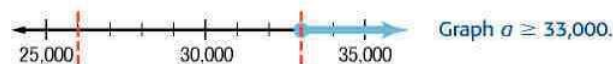
AVIATION An airplane is experiencing heavy turbulence while flying at 30,000 feet. The control tower tells the pilot that he should increase his altitude to at least 33,000 feet or decrease his altitude to no more than 26,000 feet to avoid the turbulence. Write and graph a compound inequality that describes the altitude at which the airplane should fly.

Words The pilot has been told to fly at an altitude of at least 33,000 feet or no more than 26,000 feet.

Variables Let a be the plane's altitude.

Inequality $\underbrace{\text{The plane's altitude}}_a \underbrace{\text{is at least}}_{\geq} \underbrace{33,000 \text{ feet}}_{33,000} \text{ or } \underbrace{\text{the altitude}}_a \underbrace{\text{is no more than}}_{\leq} \underbrace{26,000 \text{ feet}}_{26,000}$

Now, graph the solution set.



$a \geq 33,000$ or $a \leq 26,000$

Career Choices



Pilot

Pilots check aviation weather forecasts to choose a route and altitude that will provide the smoothest flight.

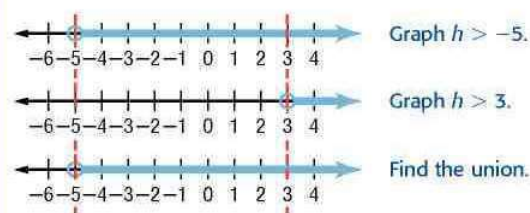
Online Research

For information about a career as a pilot, visit: www.algebra1.com/careers

Example 4 Solve and Graph a UnionSolve $-3h + 4 < 19$ or $7h - 3 > 18$. Then graph the solution set.

$$\begin{array}{rclcl}
 -3h + 4 < 19 & \text{or} & 7h - 3 > 18 \\
 -3h + 4 - 4 < 19 - 4 & & 7h - 3 + 3 > 18 + 3 \\
 -3h < 15 & & 7h > 21 \\
 \frac{-3h}{-3} > \frac{15}{-3} & & \frac{7h}{7} > \frac{21}{7} \\
 h > -5 & & h > 3
 \end{array}$$

The solution set is the union of the two graphs.



Notice that the graph of $h > -5$ contains every point in the graph of $h > 3$. So, the union is the graph of $h > -5$. The solution set is $\{h \mid h > -5\}$.

Check for Understanding**Concept Check**

- Describe the difference between a compound inequality containing *and* and a compound inequality containing *or*.
- Write *7 is less than t , which is less than 12* as a compound inequality.
- OPEN ENDED** Give an example of a compound inequality containing *and* that has no solution.

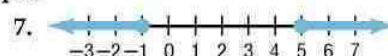
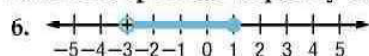
Guided Practice

Graph the solution set of each compound inequality.

4. $a \leq 6$ and $a \geq -2$

5. $y > 12$ or $y < 9$

Write a compound inequality for each graph.



Solve each compound inequality. Then graph the solution set.

8. $6 < w + 3$ and $w + 3 < 11$

9. $n - 7 \leq -5$ or $n - 7 \geq 1$

10. $3z + 1 < 13$ or $z \leq 1$

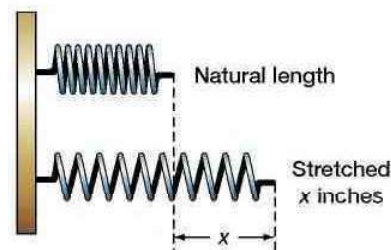
11. $-8 < x - 4 \leq -3$

12. Define a variable, write a compound inequality, and solve the following problem. Then check your solution.

Three times a number minus 7 is less than 17 and greater than 5.

Application

13. **PHYSICAL SCIENCE** According to Hooke's Law, the force F in pounds required to stretch a certain spring x inches beyond its natural length is given by $F = 4.5x$. If forces between 20 and 30 pounds, inclusive, are applied to the spring, what will be the range of the increased lengths of the stretched spring?



Practice and Apply

Homework Help

For Exercises	See Examples
14–27	1
28–45	2, 4
46–48	3

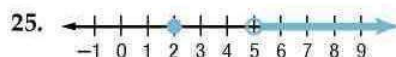
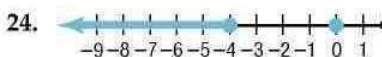
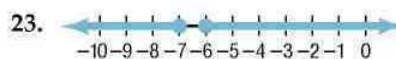
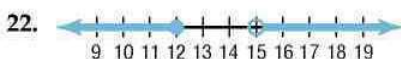
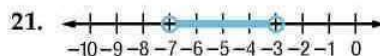
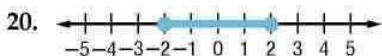
Extra Practice

See page 834.

Graph the solution set of each compound inequality.

14. $x > 5$ and $x \leq 9$ 15. $s < -7$ and $s \leq 0$ 16. $r < 6$ or $r > 6$
 17. $m \geq -4$ or $m > 6$ 18. $7 < d < 11$ 19. $-1 \leq g < 3$

Write a compound inequality for each graph.



26. **WEATHER** The Fujita Scale (F-scale) is the official classification system for tornado damage. One factor used to classify a tornado is wind speed. Use the information in the table to write an inequality for the range of wind speeds of an F3 tornado.



F-Scale Number	Rating
F0	40–72 mph
F1	73–112 mph
F2	113–157 mph
F3	158–206 mph
F4	207–260 mph
F5	261–318 mph

27. **BIOLOGY** Each type of fish thrives in a specific range of temperatures. The optimum temperatures for sharks range from 18°C to 22°C , inclusive. Write an inequality to represent temperatures where sharks will *not* thrive.

Solve each compound inequality. Then graph the solution set.

28. $k + 2 > 12$ and $k + 2 \leq 18$ 29. $f + 8 \leq 3$ and $f + 9 \geq -4$
 30. $d - 4 > 3$ or $d - 4 \leq 1$ 31. $h - 10 < -21$ or $h - 10 \leq -2$
 32. $3 < 2x - 3 < 15$ 33. $4 < 2y - 2 < 10$
 34. $3t - 7 > 5$ and $2t + 6 < 12$ 35. $8 > 5 - 3q$ and $5 - 3q > -13$
 36. $-1 + x \leq 3$ or $-x \leq -4$ 37. $3n + 11 \leq 13$ or $-3n \geq -12$
 38. $2p - 2 \leq 4p - 8 \leq 3p - 3$ 39. $3g + 12 \leq 6 + g \leq 3g - 18$
 40. $4c < 2c - 10$ or $-3c < -12$ 41. $0.5b > -6$ or $3b + 16 < -8 + b$

Define a variable, write an inequality, and solve each problem. Then check your solution.

42. Eight less than a number is no more than 14 and no less than 5.
 43. The sum of 3 times a number and 4 is between -8 and 10.
 44. The product of -5 and a number is greater than 35 or less than 10.
 45. One half a number is greater than 0 and less than or equal to 1.
 46. **HEALTH** About 20% of the time you sleep is spent in rapid eye movement (REM) sleep, which is associated with dreaming. If an adult sleeps 7 to 8 hours, how much time is spent in REM sleep?
 47. **SHOPPING** A store is offering a \$30 mail-in rebate on all color printers. Luisana is looking at different color printers that range in price from \$175 to \$260. How much can she expect to spend after the mail-in rebate?

48. **FUND-RAISING** Rashid is selling chocolates for his school's fund-raiser. He can earn prizes depending on how much he sells. So far, he has sold \$70 worth of chocolates. How much more does he need to sell to earn a prize in category D?

Sales (\$)	Prize
0–25	A
26–60	B
61–120	C
121–180	D
180+	E

49. **CRITICAL THINKING** Write a compound inequality that represents the values of x which make the following expressions *false*.
- a. $x < 5$ or $x > 8$ b. $x \leq 6$ and $x \geq 1$

HEARING For Exercises 50–52, use the following information.

Humans hear sounds with sound waves within the 20 to 20,000 hertz range.
Dogs hear sounds in the 15 to 50,000 hertz range.

50. Write a compound inequality for the hearing range of humans and one for the hearing range of dogs.
51. What is the union of the two graphs? the intersection?
52. Write an inequality or inequalities for the range of sounds that dogs can hear, but humans cannot.
53. **RESEARCH** Use the Internet or other resource to find the altitudes in miles of the layers of Earth's atmosphere, troposphere, stratosphere, mesosphere, thermosphere, and exosphere. Write inequalities for the range of altitudes for each layer.

54. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are compound inequalities used in tax tables?

Include the following in your answer:

- a description of the intervals used in the tax table shown at the beginning of the lesson, and
- a compound inequality describing the income of a head of a household paying \$7024 in taxes.



55. Ten pounds of fresh tomatoes make between 10 and 15 cups of cooked tomatoes. How many cups does one pound of tomatoes make?
- (A) between 1 and $1\frac{1}{2}$ cups (B) between 1 and 5 cups
(C) between 2 and 3 cups (D) between 2 and 4 cups
56. Solve $-7 < x + 2 < 4$.
- (A) $-5 < x < 6$ (B) $-9 < x < 2$
(C) $-5 < x < 2$ (D) $-9 < x < 6$



57. **SOLVE COMPOUND INEQUALITIES** In Lesson 6-3, you learned how to use a graphing calculator to find the values of x that make a given inequality true. You can also use this method to test compound inequalities. The words *and* and *or* can be found in the LOGIC submenu of the TEST menu of a TI-83 Plus. Use this method to solve each of the following compound inequalities using your graphing calculator:
- a. $x + 4 < -2$ or $x + 4 > 3$ b. $x - 3 \leq 5$ and $x + 6 \geq 4$

Maintain Your Skills

- Mixed Review** 58. **FUND-RAISING** A university is running a drive to raise money. A corporation has promised to match 40% of whatever the university can raise from other sources. How much must the school raise from other sources to have a total of at least \$800,000 after the corporation's donation? (Lesson 6-3)

Solve each inequality. Then check your solution. (Lesson 6-2)

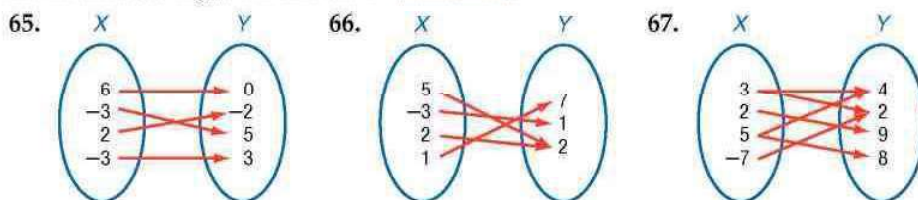
59. $18d \geq 90$ 60. $-7v < 91$ 61. $\frac{t}{13} < 13$ 62. $-\frac{3}{8}b > 9$

Solve. Assume that y varies directly as x . (Lesson 5-2)

63. If $y = -8$ when $x = -3$, find x when $y = 6$.

64. If $y = 2.5$ when $x = 0.5$, find y when $x = 20$.

Express the relation shown in each mapping as a set of ordered pairs. Then state the domain, range, and inverse. (Lesson 4-3)



Find the odds of each outcome if a die is rolled. (Lesson 2-6)

68. a number greater than 2 69. not a 3

Find each product. (Lesson 2-3)

70. $-\frac{5}{6}\left(-\frac{2}{5}\right)$ 71. $-100(4.7)$ 72. $-\frac{7}{12}\left(\frac{6}{7}\right)\left(-\frac{3}{4}\right)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each value. (To review **absolute value**, see Lesson 2-1.)

73. $|-7|$ 74. $|10|$ 75. $|-1|$ 76. $|-3.5|$
77. $|12 - 6|$ 78. $|5 - 9|$ 79. $|20 - 21|$ 80. $|3 - 18|$

Practice Quiz 2

Lessons 6-3 and 6-4

Solve each inequality. Then check your solution. (Lesson 6-3)

1. $5 - 4b > -23$ 2. $\frac{1}{2}n + 3 \geq -5$
3. $3(t + 6) < 9$ 4. $9x + 2 > 20$
5. $2m + 5 \leq 4m - 1$ 6. $a < \frac{2a - 15}{3}$

Solve each compound inequality. Then graph the solution set. (Lesson 6-4)

7. $x - 2 < 7$ and $x + 2 > 5$ 8. $2b + 5 \leq -1$ or $b - 4 \geq -4$
9. $4m - 5 > 7$ or $4m - 5 < -9$ 10. $a - 4 < 1$ and $a + 2 > 1$

6-5

Solving Open Sentences Involving Absolute Value

What You'll Learn

- Solve absolute value equations.
- Solve absolute value inequalities.

How is absolute value used in election polls?

Voters in Hamilton will vote on a new tax levy in the next election. A poll conducted before the election found that 47% of the voters surveyed were for the tax levy, 45% were against the tax levy, and 8% were undecided. The poll has a 3-point margin of error.



The margin of error means that the result may be 3 percentage points higher or lower. So, the number of people in favor of the tax levy may be as high as 50% or as low as 44%. This can be written as an inequality using absolute value.

$$|x - 47| \leq 3 \quad \text{The difference between the actual number and 47 is within 3 points.}$$

Study Tip

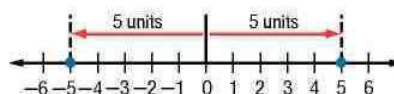
Look Back

To review **absolute value**, see Lesson 2-1.

ABSOLUTE VALUE EQUATIONS There are three types of open sentences that can involve absolute value.

$$|x| = n \quad |x| < n \quad |x| > n$$

Consider the case of $|x| = n$. $|x| = 5$ means the distance between 0 and x is 5 units.



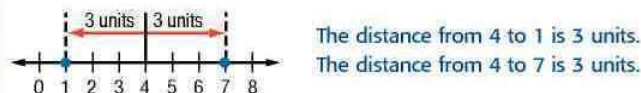
If $|x| = 5$, then $x = -5$ or $x = 5$. The solution set is $\{-5, 5\}$.

When solving equations that involve absolute value, there are two cases to consider.

Case 1 The value inside the absolute value symbols is positive.

Case 2 The value inside the absolute value symbols is negative.

Equations involving absolute value can be solved by graphing them on a number line or by writing them as a compound sentence and solving it.

Example 1 Solve an Absolute Value EquationSolve $|a - 4| = 3$.**Method 1** Graphing $|a - 4| = 3$ means that the distance between a and 4 is 3 units. To find a on the number line, start at 4 and move 3 units in either direction.The solution set is $\{1, 7\}$.**Method 2** Compound SentenceWrite $|a - 4| = 3$ as $a - 4 = 3$ or $a - 4 = -3$.**Case 1**

$$a - 4 = 3$$

$$a - 4 + 4 = 3 + 4 \quad \text{Add 4 to each side.}$$

$$a = 7 \quad \text{Simplify.}$$

Case 2

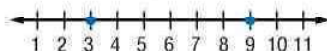
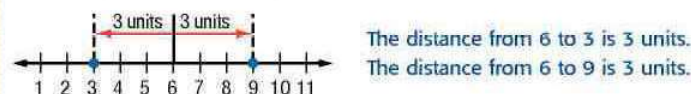
$$a - 4 = -3$$

$$a - 4 + 4 = -3 + 4 \quad \text{Add 4 to each side.}$$

$$a = 1 \quad \text{Simplify.}$$

The solution set is $\{1, 7\}$.**Study Tip****Absolute Value**Recall that $|a| = 3$ means $a = 3$ or $-a = 3$.The second equation can be written as $a = -3$.So, $|a - 4| = 3$ means $a - 4 = 3$ or $-(a - 4) = 3$. These canbe written as $a - 4 = 3$ or $a - 4 = -3$.**Example 2** Write an Absolute Value Equation

Write an equation involving absolute value for the graph.

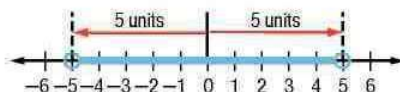
Find the point that is the same distance from 3 as the distance from 9.
The midpoint between 3 and 9 is 6.So, an equation is $|x - 6| = 3$.**CHECK** Substitute 3 and 9 into $|x - 6| = 3$.

$$|x - 6| = 3 \quad |x - 6| = 3$$

$$|3 - 6| \stackrel{?}{=} 3 \quad |9 - 6| \stackrel{?}{=} 3$$

$$|-3| \stackrel{?}{=} 3 \quad |3| \stackrel{?}{=} 3$$

$$3 = 3 \quad \checkmark \quad 3 = 3 \quad \checkmark$$

ABSOLUTE VALUE INEQUALITIES Consider the inequality $|x| < n$.
 $|x| < 5$ means that the distance from 0 to x is less than 5 units.Therefore, $x > -5$ and $x < 5$. The solution set is $\{x | -5 < x < 5\}$.

The Algebra Activity explores an inequality of the form $|x| < n$.



Algebra Activity

Absolute Value

Collect the Data

- Work in pairs. One person is the timekeeper.
- Start timing. The other person tells the timekeeper to stop timing after he or she thinks that one minute has elapsed.
- Write down the time in seconds.
- Switch places. Make a table that includes the results of the entire class.

Analyze the Data

1. Determine the error by subtracting 60 seconds from each student's time.
2. What does a negative error represent? a positive error?
3. The *absolute error* is the absolute value of the error. Since absolute value cannot be negative, the absolute error is positive. If the absolute error is 6 seconds, write two possibilities for a student's estimated time of one minute.
4. What estimates would have an absolute error less than 6 seconds?
5. Graph the responses and highlight all values such that $|60 - x| < 6$. How many guesses were within 6 seconds?

When solving inequalities of the form $|x| < n$, find the intersection of these two cases.

Case 1 The value inside the absolute value symbols is less than the positive value of n .

Case 2 The value inside the absolute value symbols is greater than the negative value of n .

Study Tip

Less Than

When an absolute value is on the left and the inequality symbol is $<$ or $\leq$, the compound sentence uses *and*.

Example 3 Solve an Absolute Value Inequality ($<$)

Solve $|t + 5| < 9$. Then graph the solution set.

Write $|t + 5| < 9$ as $t + 5 < 9$ and $t + 5 > -9$.

Case 1

$$t + 5 < 9$$

$$t + 5 - 5 < 9 - 5 \quad \text{Subtract 5 from each side.}$$

$$t < 4 \quad \text{Simplify.}$$

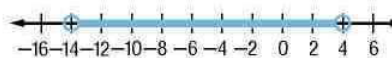
Case 2

$$t + 5 > -9$$

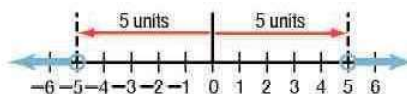
$$t + 5 - 5 > -9 - 5 \quad \text{Subtract 5 from each side.}$$

$$t > -14 \quad \text{Simplify.}$$

The solution set is $\{t \mid -14 < t < 4\}$.



Consider the inequality $|x| > n$. $|x| > 5$ means that the distance from 0 to x is greater than 5 units.



Therefore, $x < -5$ or $x > 5$. The solution set is $\{x \mid x < -5 \text{ or } x > 5\}$.



When solving inequalities of the form $|x| > n$, find the union of these two cases.

Case 1 The value inside the absolute value symbols is greater than the positive value of n .

Case 2 The value inside the absolute value symbols is less than the negative value of n .

Study Tip

Greater Than

When the absolute value is on the left and the inequality symbol is $>$ or $\geq$, the compound sentence uses *or*.

Example 4 Solve an Absolute Value Inequality ($>$)

Solve $|2x + 8| \geq 6$. Then graph the solution set.

Write $|2x + 8| \geq 6$ as $2x + 8 \geq 6$ or $2x + 8 \leq -6$.

Case 1

$$2x + 8 \geq 6$$

$$2x + 8 - 8 \geq 6 - 8 \quad \text{Subtract 8 from each side.}$$

$$2x \geq -2 \quad \text{Simplify.}$$

$$\frac{2x}{2} \geq \frac{-2}{2} \quad \text{Divide each side by 2.}$$

$$x \geq -1 \quad \text{Simplify.}$$

Case 2

$$2x + 8 \leq -6$$

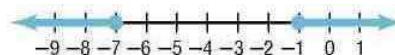
$$2x + 8 - 8 \leq -6 - 8 \quad \text{Subtract 8 from each side.}$$

$$2x \leq -14 \quad \text{Simplify.}$$

$$\frac{2x}{2} \leq \frac{-14}{2} \quad \text{Divide each side by 2.}$$

$$x \leq -7 \quad \text{Simplify.}$$

The solution set is $\{x | x \leq -7 \text{ or } x \geq -1\}$.



In general, there are three rules to remember when solving equations and inequalities involving absolute value.

Concept Summary Absolute Value Equations and Inequalities

If $|x| = n$, then $x = -n$ or $x = n$.

If $|x| < n$, then $x < n$ and $x > -n$.

If $|x| > n$, then $x > n$ or $x < -n$.

These properties are also true when $>$ or $<$ is replaced with $\geq$ or $\leq$.

Check for Understanding

Concept Check

1. Compare and contrast the solution of $|x - 2| > 6$ and the solution of $|x - 2| < 6$.
2. **OPEN ENDED** Write an absolute value inequality and graph its solution set.
3. **FIND THE ERROR** Leslie and Holly are solving $|x + 3| = 2$.

Leslie

$$\begin{aligned} x + 3 &= 2 & \text{or} & & x + 3 &= -2 \\ x + 3 - 3 &= 2 - 3 & & & x + 3 - 3 &= -2 - 3 \\ x &= -1 & & & x &= -5 \end{aligned}$$

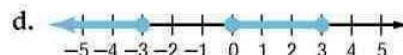
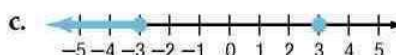
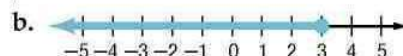
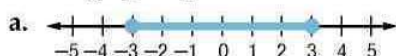
Holly

$$\begin{aligned} x + 3 &= 2 & \text{or} & & x - 3 &= 2 \\ x + 3 - 3 &= 2 - 3 & & & x - 3 + 3 &= 2 + 3 \\ x &= -1 & & & x &= 5 \end{aligned}$$

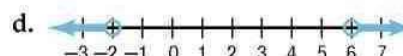
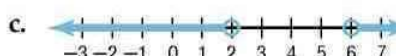
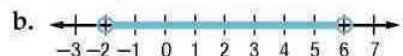
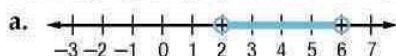
Who is correct? Explain your reasoning.

Guided Practice

4. Which graph represents the solution of $|k| \leq 3$?



5. Which graph represents the solution of $|x - 4| > 2$?



6. Express the statement in terms of an inequality involving absolute value.
Do not solve.

A jar contains 832 gumballs. Amanda's guess was within 46 pieces.

Solve each open sentence. Then graph the solution set.

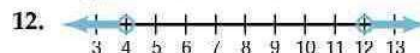
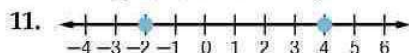
7. $|r + 3| = 10$

8. $|c - 2| < 6$

9. $|10 - w| > 15$

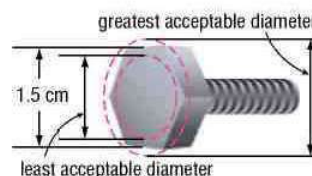
10. $|2g + 5| \geq 7$

For each graph, write an open sentence involving absolute value.



Application

13. **MANUFACTURING** A manufacturer produces bolts which must have a diameter within 0.001 centimeter of 1.5 centimeters. What are the acceptable measurements for the diameter of the bolts?



Practice and Apply

Homework Help

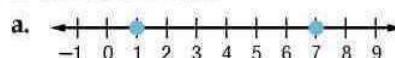
For Exercises	See Examples
14–19, 24–39, 46–51	1, 3, 4
20–23	3
40–45	2

Extra Practice

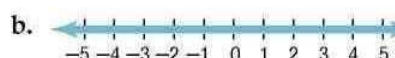
See page 834.

Match each open sentence with the graph of its solution set.

14. $|x + 5| \leq 3$



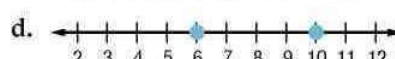
15. $|x - 4| > 4$



16. $|2x - 8| = 6$



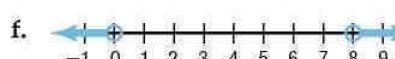
17. $|x + 3| \geq -1$



18. $|x| < 2$



19. $|8 - x| = 2$



Express each statement using an inequality involving absolute value.
Do not solve.

20. The pH of a buffered eye solution must be within 0.002 of a pH of 7.3.

21. The temperature inside a refrigerator should be within 1.5 degrees of 38°F.

22. Ramona's bowling score was within 6 points of her average score of 98.

23. The cruise control of a car set at 55 miles per hour should keep the speed within 3 miles per hour of 55.

Solve each open sentence. Then graph the solution set.

24. $|x - 5| = 8$

26. $|2p - 3| = 17$

28. $|z - 2| \leq 5$

30. $|v + 3| > 1$

32. $|3s + 2| > -7$

34. $|2n + 1| < 9$

36. $|6 - (3d - 5)| \leq 14$

38. $\left|\frac{5h + 2}{6}\right| = 7$

25. $|b + 9| = 2$

27. $|5c - 8| = 12$

29. $|t + 8| < 2$

31. $|w - 6| \geq 3$

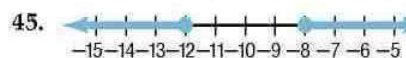
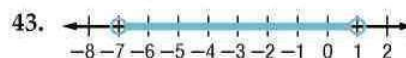
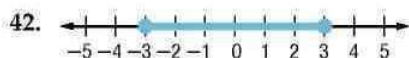
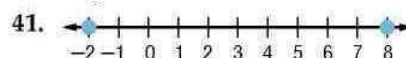
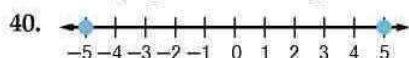
33. $|3k + 4| \geq 8$

35. $|6r + 8| < -4$

37. $|8 - (w - 1)| \leq 9$

39. $\left|\frac{2 - 3x}{5}\right| \geq 2$

For each graph, write an open sentence involving absolute value.



HEALTH For Exercises 46 and 47, use the following information.

The *average* length of a human pregnancy is 280 days. However, a healthy, full-term pregnancy can be 14 days longer or shorter.

46. Write an absolute value inequality for the length of a full-term pregnancy.

47. Solve the inequality for the length of a full-term pregnancy.

48. **FIRE SAFETY** The pressure of a typical fire extinguisher should be within 25 pounds per square inch (psi) of 195 psi. Write the range of pressures for safe fire extinguishers.

49. **HEATING** A thermostat with a 2-degree differential will keep the temperature within 2 degrees Fahrenheit of the temperature set point. Suppose your home has a thermostat with a 3-degree differential. If you set the thermostat at 68°F, what is the range of temperatures in the house?

50. **ENERGY** Use the margin of error indicated in the graph at the right to find the range of the percent of people who say protection of the environment should have priority over developing energy supplies.

51. **TIRE PRESSURE** Tire pressure is measured in pounds per square inch (psi). Tires should be kept within 2 psi of the manufacturer's recommended tire pressure. If the recommended inflation pressure for a tire is 30 psi, what is the range of acceptable pressures?

52. **CRITICAL THINKING** State whether each open sentence is *always*, *sometimes*, or *never* true.

a. $|x + 3| < -5$

b. $|x - 6| > -1$

c. $|x + 2| = 0$

More About...



Tire Pressure

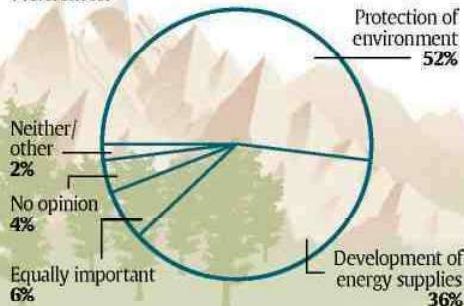
Always inflate your tires to the pressure that is recommended by the manufacturer. The pressure stamped on the tire is the *maximum* pressure and should only be used under certain circumstances.

Source: www.etires.com

USA TODAY Snapshots®

Environment first

Americans say protecting the environment should be given priority over developing U.S. energy supplies. Preferences:



Source: Gallup Poll of 1,050 adults; March 5-7, 2001. Margin of error: plus or minus 3 percentage points.

By Marcy E. Mullins, USA TODAY

More About...



Physical Science

The common name for sodium chloride is salt. Seawater is about 2.5% salt, and salt obtained by evaporating seawater is 95% to 98% pure.

Source: World Book Encyclopedia

Standardized Test Practice

A B C D

53. **PHYSICAL SCIENCE** During an experiment, Li-Cheng must add 3.0 milliliters of sodium chloride to a solution. To get accurate results, the amount of sodium chloride must be within 0.5 milliliter of the required amount. How much sodium chloride can she add and still obtain the correct results?

54. **ENTERTAINMENT** Luis Gomez is a contestant on a television game show. He must guess within \$1500 of the actual price of the car without going over in order to win the car. The actual price of the car is \$18,000. What is the range of guesses in which Luis can win the vehicle?

55. **CRITICAL THINKING** The symbol $\pm$ means *plus* or *minus*.

- If $x = 3 \pm 1.2$, what are the values of x ?
- Write $x = 3 \pm 1.2$ as an expression involving absolute value.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is absolute value used in election polls?

Include the following in your answer:

- an explanation of how to solve the inequality describing the percent of people who are against the tax levy, and
- a prediction of whether you think the tax levy will pass and why.

57. Choose the replacement set that makes $|x + 5| = 2$ true.

- (A) $\{-3, 3\}$ (B) $\{-3, -7\}$ (C) $\{2, -2\}$ (D) $\{3, -7\}$

58. What can you conclude about x if $-6 < |x| < 6$?

- (A) $-x \geq 0$ (B) $x \leq 0$ (C) $-x < 6$ (D) $-x > 6$

Maintain Your Skills

Mixed Review

59. **FITNESS** To achieve the maximum benefits from aerobic activity, your heart rate should be in your target zone. Your target zone is the range between 60% and 80% of your maximum heart rate. If Rafael's maximum heart rate is 190 beats per minute, what is his target zone? (Lesson 6-4)

Solve each inequality. Then check your solution. (Lesson 6-3)

60. $2m + 7 > 17$ 61. $-2 - 3x \geq 2$ 62. $\frac{2}{3}w - 3 \leq 7$

Find the slope and y -intercept of each equation. (Lesson 5-4)

63. $2x + y = 4$ 64. $2y - 3x = 4$ 65. $\frac{1}{2}x + \frac{3}{4}y = 0$

Solve each equation or formula for the variable specified. (Lesson 3-8)

66. $I = prt$, for r 67. $ex - 2y = 3z$, for x 68. $\frac{a+5}{3} = 7x$, for x

Find each sum or difference. (Lesson 2-2)

69. $-13 + 8$ 70. $-13.2 - 6.1$ 71. $-4.7 - (-8.9)$

Name the property illustrated by each statement. (Lesson 1-6)

72. $10x + 10y = 10(x + y)$ 73. $(2 + 3)a + 7 = 5a + 7$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Graph each equation.

(To review graphing linear equations, see Lesson 4-5.)

74. $y = 3x + 4$ 75. $y = -2$ 76. $x + y = 3$
77. $y - 2x = -1$ 78. $2y - x = -6$ 79. $2(x + y) = 10$

6-6

Graphing Inequalities in Two Variables

What You'll Learn

- Graph inequalities on the coordinate plane.
- Solve real-world problems involving linear inequalities.

Vocabulary

- half-plane
- boundary

How are inequalities used in budgets?

Hannah allots up to \$30 a month for lunch on school days. On most days, she brings her lunch. She can also buy lunch at the cafeteria or at a fast-food restaurant. She spends an average of \$3 a day at the cafeteria and an average of \$4 a day at a restaurant. How many times a month can Hannah buy her lunch and remain within her budget?

My Monthly Budget	
Lunch (school days)	\$30
Entertainment	\$55
Clothes	\$50
Fuel	\$60

Let x represent the number of days she buys lunch at the cafeteria, and let y represent the number of days she buys lunch at a restaurant. Then the following inequality can be used to represent the situation.

$$\underbrace{\text{The cost of eating in the cafeteria}}_{3x} \text{ plus } \underbrace{\text{the cost of eating in a restaurant}}_{4y} \text{ is less than or equal to } \underbrace{\$30}_{30}$$

$$3x + 4y \leq 30$$

There are many solutions of this inequality.

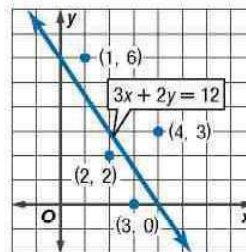
GRAPH LINEAR INEQUALITIES Like a linear equation in two variables, the solution set of an inequality in two variables is graphed on a coordinate plane. The solution set of an inequality in two variables is the set of all ordered pairs that satisfy the inequality.

Example 1 Ordered Pairs that Satisfy an Inequality

From the set $\{(1, 6), (3, 0), (2, 2), (4, 3)\}$, which ordered pairs are part of the solution set for $3x + 2y < 12$?

Use a table to substitute the x and y values of each ordered pair into the inequality.

x	y	$3x + 2y < 12$	True or False
1	6	$3(1) + 2(6) < 12$ $15 < 12$	false
3	0	$3(3) + 2(0) < 12$ $9 < 12$	true
2	2	$3(2) + 2(2) < 12$ $10 < 12$	true
4	3	$3(4) + 2(3) < 12$ $18 < 12$	false



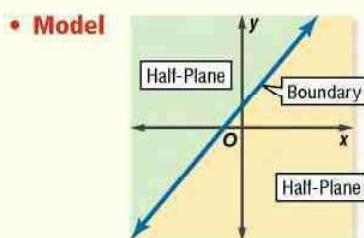
The ordered pairs $\{(3, 0), (2, 2)\}$ are part of the solution set of $3x + 2y < 12$. In the graph, notice the location of the two ordered pairs that are solutions for $3x + 2y < 12$ in relation to the line.

The solution set for an inequality in two variables contains many ordered pairs when the domain and range are the set of real numbers. The graphs of all of these ordered pairs fill a region on the coordinate plane called a **half-plane**. An equation defines the **boundary** or edge for each half-plane.

Key Concept

Half-Planes and Boundaries

- **Words** Any line in the plane divides the plane into two regions called half-planes. The line is called the boundary of each of the two half-planes.



Study Tip

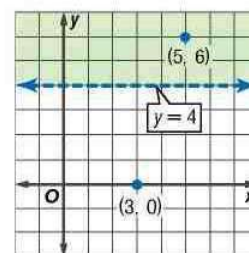
Dashed Line

- Like a circle on a number line, a dashed line on a coordinate plane indicates that the boundary is *not* part of the solution set.

Solid Line

- Like a dot on a number line, a solid line on a coordinate plane indicates that the boundary *is* included.

Consider the graph of $y > 4$. First determine the boundary by graphing $y = 4$, the equation you obtain by replacing the inequality sign with an equals sign. Since the inequality involves y -values greater than 4, but not equal to 4, the line should be dashed. The boundary divides the coordinate plane into two half-planes.



To determine which half-plane contains the solution, choose a point from each half-plane and test it in the inequality.

Try (3, 0).

$$y > 4 \quad y = 0$$

$$0 > 4 \quad \text{false}$$

Try (5, 6).

$$y > 4 \quad y = 6$$

$$6 > 4 \quad \text{true}$$

The half-plane that contains (5, 6) contains the solution. Shade that half-plane.

Example 2 Graph an Inequality

Graph $y - 2x \leq -4$.

Step 1 Solve for y in terms of x .

$$y - 2x \leq -4 \quad \text{Original inequality}$$

$$y - 2x + 2x \leq -4 + 2x \quad \text{Add } 2x \text{ to each side.}$$

$$y \leq 2x - 4 \quad \text{Simplify.}$$

Step 2 Graph $y = 2x - 4$. Since $y \leq 2x - 4$ means $y < 2x - 4$ or $y = 2x - 4$, the boundary is included in the solution set. The boundary should be drawn as a solid line.

(continued on the next page)

Study Tip

Origin as the Test Point

Use the origin as a standard test point because the values are easy to substitute into the inequality.

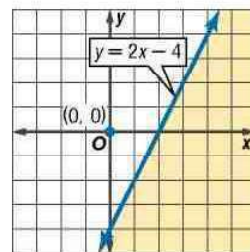
Step 3 Select a point in one of the half-planes and test it. Let's use $(0, 0)$.

$$y \leq 2x - 4 \quad \text{Original inequality}$$

$$0 \leq 2(0) - 4 \quad x = 0, y = 0$$

$$0 \leq -4 \quad \text{false}$$

Since the statement is false, the half-plane containing the origin is not part of the solution. Shade the other half-plane.



CHECK Test a point in the other half plane, for example, $(3, -3)$.

$$y \leq 2x - 4 \quad \text{Original inequality}$$

$$-3 \leq 2(3) - 4 \quad x = 3, y = -3$$

$$-3 \leq 2 \quad \checkmark$$

Since the statement is true, the half-plane containing $(3, -3)$ should be shaded. The graph of the solution is correct.

SOLVE REAL-WORLD PROBLEMS When solving real-world inequalities, the domain and range of the inequality are often restricted to nonnegative numbers or whole numbers.

Example 3 Write and Solve an Inequality

ADVERTISING Rosa Padilla sells radio advertising in 30-second and 60-second time slots. During every hour, there are up to 15 minutes available for commercials. How many commercial slots can she sell for one hour of broadcasting?

Step 1 Let x equal the number of 30-second commercials. Let y equal the number of 60-second or 1-minute commercials. Write an open sentence representing this situation.

$$\underbrace{\frac{1}{2} \text{ min}}_{\frac{1}{2}} \underbrace{\text{times}}_{\cdot} \underbrace{\text{the number of 30-s commercials}}_x \underbrace{\text{plus}}_{+} \underbrace{\text{the number of 1-min commercials}}_y \underbrace{\text{is up to}}_{\leq} \underbrace{15 \text{ min.}}_{15}$$

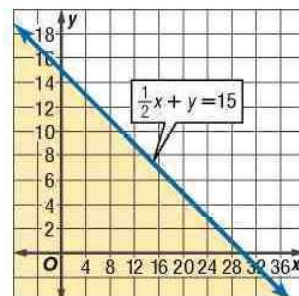
Step 2 Solve for y in terms of x .

$$\frac{1}{2}x + y \leq 15 \quad \text{Original inequality}$$

$$\frac{1}{2}x + y - \frac{1}{2}x \leq 15 - \frac{1}{2}x \quad \text{Subtract } \frac{1}{2}x \text{ from each side.}$$

$$y \leq 15 - \frac{1}{2}x \quad \text{Simplify.}$$

Step 3 Since the open sentence includes the equation, graph $y = 15 - \frac{1}{2}x$ as a solid line. Test a point in one of the half-planes, for example $(0, 0)$. Shade the half-plane containing $(0, 0)$ since $0 \leq 15 - \frac{1}{2}(0)$ is true.



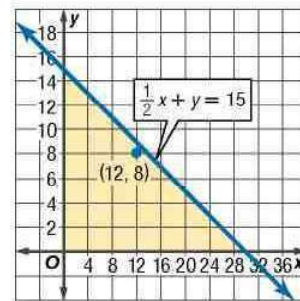
Advertising

A typical one-hour program on television contains 40 minutes of the program and 20 minutes of commercials. During peak periods, a 30-second commercial can cost an average of \$2.3 million.

Source: www.superbowl-ads.com

Step 4 Examine the solution.

- Rosa cannot sell a negative number of commercials. Therefore, the domain and range contain only nonnegative numbers.
- She also cannot sell half of a commercial. Thus, only points in the shaded half-plane whose x - and y -coordinates are whole numbers are possible solutions.



One solution is (12, 8). This represents twelve 30-second commercials and eight 60-second commercials in a one hour period.

Check for Understanding

Concept Check

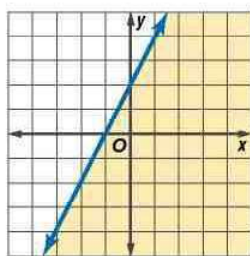
1. Compare and contrast the graph of $y = x + 2$ and the graph of $y < x + 2$.
2. **OPEN ENDED** Write an inequality in two variables and graph it.
3. Explain why it is usually only necessary to test one point when graphing an inequality.

Guided Practice

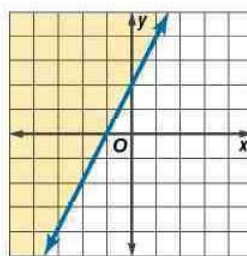
Determine which ordered pairs are part of the solution set for each inequality.

4. $y \leq x + 1$, $\{(-1, 0), (3, 2), (2, 5), (-2, 1)\}$
5. $y > 2x$, $\{(2, 6), (0, -1), (3, 5), (-1, -2)\}$
6. Which graph represents $y - 2x \geq 2$?

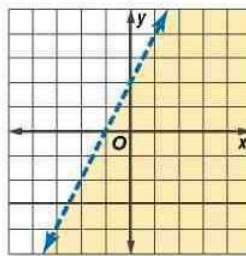
a.



b.



c.



Graph each inequality.

7. $y \geq 4$
8. $y \leq 2x - 3$
9. $4 - 2x < -2$
10. $1 - y > x$

Application

11. **ENTERTAINMENT** Coach Riley wants to take her softball team out for pizza and soft drinks after the last game of the season. She doesn't want to spend more than \$60. Write an inequality that represents this situation and graph the solution set.



Practice and Apply

Homework Help

For Exercises	See Examples
12–19	1
20–37	2
38–44	3

Extra Practice

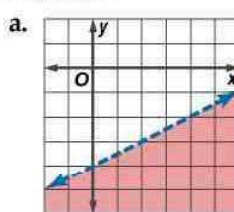
See page 835.

Determine which ordered pairs are part of the solution set for each inequality.

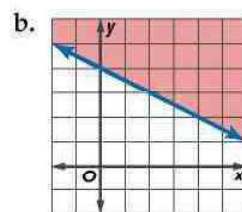
12. $y \leq 3 - 2x$, $\{(0, 4), (-1, 3), (6, -8), (-4, 5)\}$
13. $y < 3x$, $\{(-3, 1), (-3, 2), (1, 1), (1, 2)\}$
14. $x + y < 11$, $\{(5, 7), (-13, 10), (4, 4), (-6, -2)\}$
15. $2x - 3y > 6$, $\{(3, 2), (-2, -4), (6, 2), (5, 1)\}$
16. $4y - 8 \geq 0$, $\{(5, -1), (0, 2), (2, 5), (-2, 0)\}$
17. $3x + 4y < 7$, $\{(1, 1), (2, -1), (-1, 1), (-2, 4)\}$
18. $|x - 3| \geq y$, $\{(6, 4), (-1, 8), (-3, 2), (5, 7)\}$
19. $|y + 2| < x$, $\{(2, -4), (-1, -5), (6, -7), (0, 0)\}$

Match each inequality with its graph.

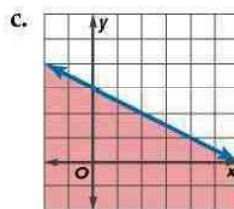
20. $2y + x \leq 6$



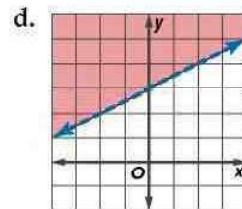
21. $\frac{1}{2}x - y > 4$



22. $y > 3 + \frac{1}{2}x$



23. $4y + 2x \geq 16$



24. Is the point $A(2, 3)$ on, above, or below the graph of $-2x + 3y = 5$?

25. Is the point $B(0, 1)$ on, above, or below the graph of $4x - 3y = 4$?

Graph each inequality.

- | | | | |
|---------------------|---------------------|-----------------------|-------------------------------|
| 26. $y < -3$ | 27. $x \geq 2$ | 28. $5x + 10y > 0$ | 29. $y < x$ |
| 30. $2y - x \leq 6$ | 31. $6x + 3y > 9$ | 32. $3y - 4x \geq 12$ | 33. $y \leq -2x - 4$ |
| 34. $8x - 6y < 10$ | 35. $3x - 1 \geq y$ | 36. $3(x + 2y) > -18$ | 37. $\frac{1}{2}(2x + y) < 2$ |

POSTAGE For Exercises 38 and 39, use the following information.

The U.S. Postal Service defines a large package as having the length of its longest side plus the distance around its thickest part less than or equal to 108 inches.

38. Write an inequality that represents this situation.

39. Are there any restrictions on the domain or range?

 **Online Research Data Update** What are the current postage rates and regulations? Visit www.algebra1.com/data_update to learn more.

SHIPPING For Exercises 40 and 41, use the following information.

A delivery truck is transporting televisions and microwaves to an appliance store. The weight limit for the truck is 4000 pounds. The televisions weigh 77 pounds, and the microwaves weigh 55 pounds.

40. Write an inequality for this situation.

41. Will the truck be able to deliver 35 televisions and 25 microwaves at once?

WebQuest

A linear inequality can be used to represent trends in Olympic times. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

FALL DANCE For Exercises 42–44, use the following information.

Tickets for the fall dance are \$5 per person or \$8 for couples. In order to cover expenses, at least \$1200 worth of tickets must be sold.

42. Write an inequality that represents this situation.
43. Graph the inequality.
44. If 100 single tickets and 125 couple tickets are sold, will the committee cover its expenses?

45. **CRITICAL THINKING** Graph the intersection of the graphs of $y \leq x - 1$ and $y \geq -x$.

46. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are inequalities used in budgets?

Include the following in your answer:

- an explanation of the restrictions placed on the domain and range of the inequality used to describe the number of times Hannah can buy her lunch, and
- three possible solutions of the inequality.

Standardized Test Practice

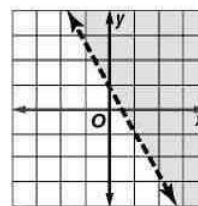
A B C D

47. Which ordered pair is *not* a solution of $y - 2x < -5$?

- (A) (2, -2) (B) (-1, -8) (C) (4, 1) (D) (5, 6)

48. Which inequality is represented by the graph at the right?

- (A) $2x + y < 1$ (B) $2x + y > 1$
(C) $2x + y \leq 1$ (D) $2x + y \geq 1$



Maintain Your Skills

Mixed Review

Solve each open sentence. Then graph the solution set. (Lesson 6-5)

49. $|3 + 2t| = 11$ 50. $|x + 8| < 6$ 51. $|2y + 5| \geq 3$

Solve each compound inequality. Then graph the solution. (Lesson 6-4)

52. $y + 6 > -1$ and $y - 2 < 4$ 53. $m + 4 < 2$ or $m - 2 > 1$

State whether each percent of change is a percent of *increase* or *decrease*.

Then find the percent of change. Round to the nearest whole percent. (Lesson 3-7)

54. original: 200 55. original: 100 56. original: 53
new: 172 new: 142 new: 75

Solve each equation. (Lesson 3-4)

57. $\frac{d-2}{3} = 7$ 58. $3n + 6 = -15$ 59. $35 + 20h = 100$

Simplify. (Lesson 2-4)

60. $\frac{-64}{4}$ 61. $\frac{27c}{-9}$ 62. $\frac{12a-14b}{-2}$ 63. $\frac{18y-9}{3}$



Graphing Calculator Investigation

A Follow-Up of Lesson 6-6

Graphing Inequalities

You can use a TI-83 Plus graphing calculator to investigate the graphs of inequalities. Since graphing calculators only shade between two functions, enter a lower boundary as well as an upper boundary for each inequality.

Graph two different inequalities on your graphing calculator.

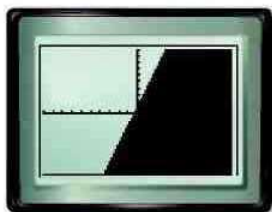
Step 1 Graph $y \leq 3x + 1$.

- Clear all functions from the Y= list.

KEYSTROKES: **Y=** **CLEAR**

- Graph $y \leq 3x + 1$ in the standard window.

KEYSTROKES: **2nd** **DRAW** 7 **(←)** 10 **,** 3
X,T,θ,n **+** 1 **)** **ENTER**



The lower boundary is Ymin or -10 . The upper boundary is $y = 3x + 1$. All ordered pairs for which y is less than or equal to $3x + 1$ lie below or on the line and are solutions.

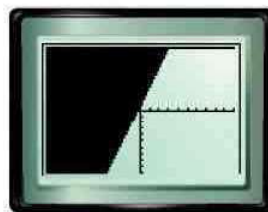
Step 2 Graph $y - 3x \geq 1$.

- Clear the drawing that is currently displayed.

KEYSTROKES: **2nd** **DRAW** 1

- Rewrite $y - 3x \geq 1$ as $y \geq 3x + 1$ and graph it.

KEYSTROKES: **2nd** **DRAW** 7 3 **X,T,θ,n**
+ 1 **,** 10 **)** **ENTER**



This time, the lower boundary is $y = 3x + 1$. The upper boundary is Ymax or 10. All ordered pairs for which y is greater than or equal to $3x + 1$ lie above or on the line and are solutions.

Exercises

- Compare and contrast the two graphs shown above.
- Graph the inequality $y \geq -2x + 4$ in the standard viewing window.
 - What functions do you enter as the lower and upper boundaries?
 - Using your graph, name four solutions of the inequality.
- Suppose student movie tickets cost \$4 and adult movie tickets cost \$8. You would like to buy at least 10 tickets, but spend no more than \$80.
 - Let x = number of student tickets and y = number of adult tickets. Write two inequalities, one representing the total number of tickets and the other representing the total cost of the tickets.
 - Which inequalities would you use as the lower and upper bounds?
 - Graph the inequalities. Use the viewing window $[0, 20]$ scl: 1 by $[0, 20]$ scl: 1.
 - Name four possible combinations of student and adult tickets.



www.algebra1.com/other_calculator_keystrokes

Vocabulary and Concept Check

Addition Property of Inequalities (p. 318)
 boundary (p. 353)
 compound inequality (p. 339)
 Division Property of Inequalities (p. 327)

half-plane (p. 353)
 intersection (p. 339)
 Multiplication Property of
 Inequalities (p. 325)

set-builder notation (p. 319)
 Subtraction Property of
 Inequalities (p. 319)
 union (p. 340)

Choose the letter of the term that best matches each statement, algebraic expression, or algebraic sentence.

- $\{w \mid w \geq -14\}$
- If $x \leq y$, then $-5x \geq -5y$.
- $p > -5$ and $p \leq 0$
- If $a < b$, then $a + 2 < b + 2$.
- the graph on one side of a boundary
- If $s \geq t$, then $s - 7 \geq t - 7$.
- $g \geq 7$ or $g < 2$
- If $m > n$, then $\frac{m}{7} > \frac{n}{7}$.

- Addition Property of Inequalities
- Division Property of Inequalities
- half-plane
- intersection
- Multiplication Property of Inequalities
- set-builder notation
- Subtraction Property of Inequalities
- union

Lesson-by-Lesson Review

6-1 Solving Inequalities by Addition and Subtraction

See pages
318–323.

Concept Summary

- If any number is added to each side of a true inequality, the resulting inequality is also true.
- If any number is subtracted from each side of a true inequality, the resulting inequality is also true.

Examples Solve each inequality.

1 $f + 9 \leq -23$

$$\begin{array}{ll} f + 9 \leq -23 & \text{Original inequality} \\ f + 9 - 9 \leq -23 - 9 & \text{Subtract} \\ f \leq -32 & \text{Simplify} \end{array}$$

The solution set is $\{f \mid f \leq -32\}$.

2 $v - 19 > -16$

$$\begin{array}{ll} v - 19 > -16 & \text{Original inequality} \\ v - 19 + 19 > -16 + 19 & \text{Add} \\ v > 3 & \text{Simplify} \end{array}$$

The solution set is $\{v \mid v > 3\}$.

Exercises Solve each inequality. Then check your solution, and graph it on a number line. See Examples 1–5 on pages 318–320.

9. $c + 51 > 32$

10. $r + 7 > -5$

11. $w - 14 \leq 23$

12. $a - 6 > -10$

13. $-0.11 \geq n - (-0.04)$

14. $2.3 < g - (-2.1)$

15. $7h \leq 6h - 1$

16. $5b > 4b + 5$

17. Define a variable, write an inequality, and solve the problem. Then check your solution. *Twenty-one is no less than the sum of a number and negative two.*



6-2 Solving Inequalities by Multiplication and Division

See pages
325–331.

Concept Summary

- If each side of a true inequality is multiplied or divided by the same positive number, the resulting inequality is also true.
- If each side of a true inequality is multiplied or divided by the same negative number, the direction of the inequality must be *reversed*.

Examples Solve each inequality.

1 $-14g \geq 126$

$$-14g \geq 126 \quad \text{Original inequality}$$

$$\frac{-14g}{-14} \leq \frac{126}{-14} \quad \text{Divide and change } \geq \text{ to } \leq.$$

$$g \leq -9 \quad \text{Simplify.}$$

The solution set is $\{g \mid g \leq -9\}$.

2 $\frac{3}{4}d < 15$

$$\frac{3}{4}d < 15 \quad \text{Original inequality}$$

$$\left(\frac{4}{3}\right)\frac{3}{4}d < \left(\frac{4}{3}\right)15 \quad \text{Multiply each side by } \frac{4}{3}.$$

$$d < 20 \quad \text{Simplify.}$$

The solution set is $\{d \mid d < 20\}$.

Exercises Solve each inequality. Then check your solution.

See Examples 1–5 on pages 326–328.

18. $15v > 60$

19. $12r \leq 72$

20. $-15z \geq -75$

21. $-9m < 99$

22. $\frac{b}{-12} \leq 3$

23. $\frac{d}{-13} > -5$

24. $\frac{2}{3}w > -22$

25. $\frac{3}{5}p \leq -15$

26. Define a variable, write an inequality, and solve the problem. Then check your solution. *Eighty percent of a number is greater than or equal to 24.*

6-3 Solving Multi-Step Inequalities

See pages
332–337.

Concept Summary

- Multi-step inequalities can be solved by undoing the operations.
- Remember to reverse the inequality sign when multiplying or dividing each side by a negative number.
- When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example Solve $4(n - 1) < 7n + 8$.

$$4(n - 1) < 7n + 8 \quad \text{Original inequality}$$

$$4n - 4 < 7n + 8 \quad \text{Distributive Property}$$

$$4n - 4 - 7n < 7n + 8 - 7n \quad \text{Subtract } 7n \text{ from each side.}$$

$$-3n - 4 < 8 \quad \text{Simplify.}$$

$$-3n - 4 + 4 < 8 + 4 \quad \text{Add 4 to each side.}$$

$$-3n < 12 \quad \text{Simplify.}$$

$$\frac{-3n}{-3} > \frac{12}{-3} \quad \text{Divide each side by } -3 \text{ and change } < \text{ to } >.$$

$$n > -4 \quad \text{Simplify.}$$

The solution set is $\{n \mid n > -4\}$.

Exercises Solve each inequality. Then check your solution.

See Examples 1–5 on pages 332–334.

27. $-4h + 7 > 15$ 28. $5 - 6n > -19$ 29. $-5x + 3 < 3x + 19$
 30. $15b - 12 > 7b + 60$ 31. $-5(q + 12) < 3q - 4$ 32. $7(g + 8) < 3(g + 2) + 4g$
 33. $\frac{2(x + 2)}{3} \geq 4$ 34. $\frac{1 - 7n}{5} > 10$
 35. Define a variable, write an inequality, and solve the problem. Then check your solution. *Two thirds of a number decreased by 27 is at least 9.*

6-4 Solving Compound Inequalities

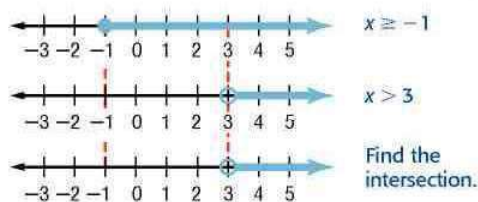
See pages
339–344.

Concept Summary

- The solution of a compound inequality containing *and* is the intersection of the graphs of the two inequalities.
- The solution of a compound inequality containing *or* is the union of the graphs of the two inequalities.

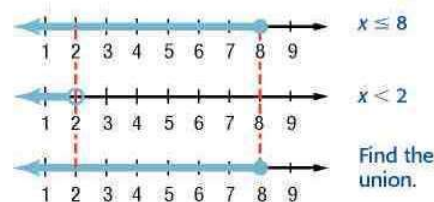
Examples Graph the solution set of each compound inequality.

1 $x \geq -1$ and $x > 3$



The solution set is $\{x \mid x > 3\}$.

2 $x \leq 8$ or $x < 2$



The solution set is $\{x \mid x \leq 8\}$.

Exercises Solve each compound inequality. Then graph the solution set.

See Examples 1–4 on pages 339–341.

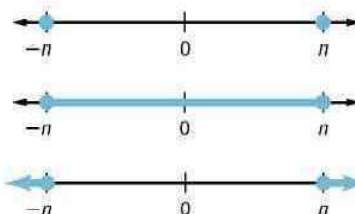
36. $-1 < p + 3 < 5$ 37. $-3 < 2k - 1 < 5$ 38. $3w + 8 < 2$ or $w + 12 > 2 - w$
 39. $a - 3 \leq 8$ or $a + 5 \geq 21$ 40. $m + 8 < 4$ and $3 - m < 5$ 41. $10 - 2y > 12$ and $7y < 4y + 9$

6-5 Solving Open Sentences Involving Absolute Value

See pages
345–351.

Concept Summary

- If $|x| = n$, then $x = -n$ or $x = n$.
- If $|x| < n$, then $x > -n$ and $x < n$.
- If $|x| > n$, then $x < -n$ or $x > n$.



- Extra Practice, see pages 833–835.
- Mixed Problem Solving, see page 858.

Example Solve $|x + 6| = 15$.

$$|x + 6| = 15$$

$$x + 6 = 15 \quad \text{or} \quad x + 6 = -15$$

$$x + 6 - 6 = 15 - 6 \quad x + 6 - 6 = -15 - 6$$

$$x = 9 \quad x = -21$$

The solution set is $\{-21, 9\}$.

Exercises Solve each open sentence. Then graph the solution set.

See Examples 1, 3, and 4 on pages 346–348.

42. $|w - 8| = 12$ 43. $|q + 5| = 2$ 44. $|h + 5| > 7$ 45. $|w + 8| \geq 1$

46. $|r + 10| < 3$ 47. $|t + 4| \leq 3$ 48. $|2x + 5| < 4$ 49. $|3d + 4| < 8$

6-6 Graphing Inequalities in Two Variables

See pages
352–357.

Concept Summary

- To graph an inequality in two variables:
 - Step 1** Determine the boundary and draw a dashed or solid line.
 - Step 2** Select a test point. Test that point.
 - Step 3** Shade the half-plane that contains the solution.

Example Graph $y \geq x - 2$.

Since the boundary is included in the solution, draw a solid line.

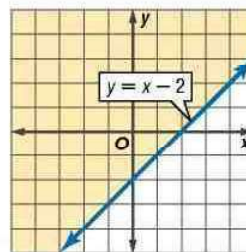
Test the point $(0, 0)$.

$$y \geq x - 2 \quad \text{Original inequality}$$

$$0 \geq 0 - 2 \quad x = 0, y = 0$$

$$0 \geq -2 \quad \text{true}$$

The half plane that contains $(0, 0)$ should be shaded.



Exercises Determine which ordered pairs are part of the solution set for each inequality. See Example 1 on page 352.

50. $3x + 2y < 9$, $\{(1, 3), (3, 2), (-2, 7), (-4, 11)\}$

51. $5 - y \geq 4x$, $\{(2, -5), (\frac{1}{2}, 7), (-1, 6), (-3, 20)\}$

52. $\frac{1}{2}y \leq 6 - x$, $\{(-4, 15), (5, 1), (3, 8), (-2, 25)\}$

53. $-2x < 8 - y$, $\{(5, 10), (3, 6), (-4, 0), (-3, 6)\}$

Graph each inequality. See Example 2 on pages 353 and 354.

54. $y - 2x < -3$ 55. $x + 2y \geq 4$ 56. $y \leq 5x + 1$ 57. $2x - 3y > 6$

Chapter 6

Practice Test

Vocabulary and Concepts

1. Write the set of all numbers t such that t is greater than or equal to 17 in set-builder notation.
2. Show how to solve $6(a + 5) < 2a + 8$. Justify your work.
3. **OPEN ENDED** Give an example of a compound inequality that is an intersection and an example of a compound inequality that is a union.
4. Compare and contrast the graphs of $|x| \leq 3$ and $|x| \geq 3$.

Skills and Applications

Solve each inequality. Then check your solution.

5. $-23 \geq g - 6$
6. $9p < 8p - 18$
7. $d - 5 < 2d - 14$
8. $\frac{7}{8}w \geq -21$
9. $-22b \geq 99$
10. $4m - 11 \geq 8m + 7$
11. $-3(k - 2) > 12$
12. $\frac{f-5}{3} > -3$
13. $0.3(y - 4) \leq 0.8(0.2y + 2)$
14. **REAL ESTATE** A homeowner is selling her house. She must pay 7% of the selling price to her real estate agent after the house is sold. To the nearest dollar, what must be the selling price of her house to have at least \$110,000 after the agent is paid?
15. Solve $6 + |r| = 3$.
16. Solve $|d| > -2$.

Solve each compound inequality. Then graph the solution set.

17. $r + 3 > 2$ and $4r < 12$
18. $3n + 2 \geq 17$ or $3n + 2 \leq -1$
19. $9 + 2p > 3$ and $-13 > 8p + 3$
20. $|2a - 5| < 7$
21. $|7 - 3s| \geq 2$
22. $|7 - 5z| > 3$

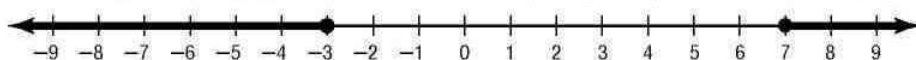
Define a variable, write an inequality, and solve each problem. Then check your solution.

23. One fourth of a number is no less than -3 .
24. Three times a number subtracted from 14 is less than two.
25. Five less than twice a number is between 13 and 21.
26. **TRAVEL** Megan's car gets between 18 and 21 miles per gallon of gasoline. If her car's tank holds 15 gallons, what is the range of distance that Megan can drive her car on one tank of gasoline?

Graph each inequality.

27. $y \geq 3x - 2$
28. $2x + 3y < 6$
29. $x - 2y > 4$

30. **STANDARDIZED TEST PRACTICE** Which inequality is represented by the graph?



- (A) $|x - 2| \leq 5$ (B) $|x - 2| \geq 5$ (C) $|x + 2| \leq 5$ (D) $|x + 2| \geq 5$

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Which of the following is a correct statement?

(Lesson 2-1)

- (A) $-\frac{9}{3} > \frac{3}{9}$ (B) $-\frac{3}{9} > -\frac{9}{3}$
 (C) $-\frac{3}{9} < -\frac{9}{3}$ (D) $\frac{9}{3} < \frac{3}{9}$

2. $(-6)(-7) =$ (Lesson 2-3)

- (A) -42 (B) -13
 (C) 13 (D) 42

3. A cylindrical can has a volume of 5625π cubic centimeters. Its height is 25 centimeters. What is the radius of the can? Use the formula $V = \pi r^2 h$. (Lessons 2-8 and 3-8)

- (A) 4.8 cm (B) 7.5 cm
 (C) 15 cm (D) 47.1 cm

4. A furnace repair service charged a customer \$80 for parts and \$65 per hour worked. The bill totaled \$177.50. About how long did the repair technician work on the furnace?

(Lessons 3-1 and 3-4)

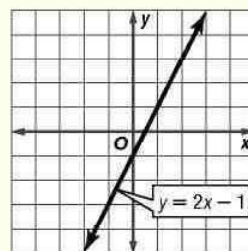
- (A) 0.5 hour (B) 1.5 hours
 (C) 2 hours (D) 4 hours

5. The formula $P = \frac{4(220 - A)}{5}$ determines the recommended maximum pulse rate P during exercise for a person who is A years old. Cameron is 15 years old. What is his recommended maximum pulse rate during exercise? (Lesson 3-8)

- (A) 162 (B) 164
 (C) 173 (D) 263

6. The graph of the function $y = 2x - 1$ is shown. If the graph is translated 3 units up, which equation will best represent the new line?

(Lesson 4-2)



- (A) $y = 2x + 2$ (B) $y = 2x - 3$
 (C) $y = 2x + 3$ (D) $y = 2x - 4$

7. The table shows a set of values for x and y . Which equation best represents this set of data? (Lesson 4-8)

x	-4	-1	2	5	8
y	-16	-4	8	20	32

- (A) $y = 3x - 4$ (B) $y = 3x + 2$
 (C) $y = 2x - 10$ (D) $y = 4x$

8. Ali's grade depends on 4 test scores. On the first 3 tests, she earned scores of 78, 82, and 75. She wants to average at least 80. Which inequality can she use to find the score x that she needs on the fourth test in order to earn a final grade of at least 80? (Lesson 6-3)

- (A) $\frac{78 + 82 + 75 + x}{3} \geq 80$
 (B) $\frac{78 + 82 + 75 + x}{4} \geq 80$
 (C) $\frac{78 + 82 + 75 - x}{4} \geq 80$
 (D) $\frac{78 + 82 + 75 + x}{4} \leq 80$

9. Which inequality is represented by the graph? (Lesson 6-4)



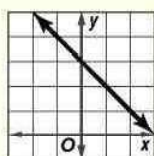
- (A) $-2 < x < 3$ (B) $-2 < x \leq 3$
 (C) $-2 \leq x < 3$ (D) $-2 \leq x \leq 3$

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. A die is rolled. What are the odds of rolling a number less than 5? (Lesson 2-6)
11. A car is traveling at an average speed of 54 miles per hour. How many minutes will it take the car to travel 117 miles? (Lesson 2-4)
12. The price of a tape player was cut from \$48 to \$36. What was the percent of decrease? (Lesson 3-7)

13. Write an equation in slope-intercept form that describes the graph. (Lesson 5-4)



14. A line is parallel to the graph of the equation $\frac{1}{3}y = \frac{2}{3}x - 1$. What is the slope of the parallel line? (Lessons 5-4 and 5-6)
15. Solve $\frac{1}{2}(10x - 8) - 3(x - 1) \geq 15$ for x . (Lesson 6-3)
16. Find all values of x that make the inequality $|x - 3| > 5$ true. (Lesson 6-5)



Test-Taking Tip

Questions 13 and 14

- Know the slope-intercept form of linear equations: $y = mx + b$.
- Understand the definition of slope.
- Recognize the relationships between the slopes of parallel lines and between the slopes of perpendicular lines.

17. Graph the equation $y = -2x + 4$ and indicate which region represents $y < -2x + 4$. (Lesson 6-6)

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- (A) the quantity in Column A is greater,
- (B) the quantity in Column B is greater,
- (C) the two quantities are equal, or
- (D) the relationship cannot be determined from the information given.

	Column A	Column B
18.	$\sqrt{68}$	9
(Lesson 2-7)		
19.	$x > 5$ or $x < -7$ $-3 < y < 4$	
	$ x $	$ y $
(Lesson 6-4)		

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

20. The Carlson family is building a house on a lot that is 91 feet long and 158 feet wide. (Lessons 6-1, 6-2, and 6-4)
 - a. Town law states that the sides of a house cannot be closer than 10 feet to the edges of a lot. Write an inequality for the possible lengths of the Carlson family's house, and solve the inequality.
 - b. The Carlson family wants their house to be at least 2800 square feet and no more than 3200 square feet. They also want their house to have the maximum possible length. Write an inequality for the possible widths of their house, and solve the inequality. Round your answer to the nearest whole number of feet.

Chapter

7

Solving Systems of Linear Equations and Inequalities

What You'll Learn

- **Lesson 7-1** Solve systems of linear equations by graphing.
- **Lessons 7-2 through 7-4** Solve systems of linear equations algebraically.
- **Lesson 7-5** Solve systems of linear inequalities by graphing.

Key Vocabulary

- system of equations (p. 369)
- substitution (p. 376)
- elimination (p. 382)
- system of inequalities (p. 394)

Why It's Important

Business decision makers often use systems of linear equations to model a real-world situation in order to predict future events. Being able to make an accurate prediction helps them plan and manage their businesses.

Trends in the travel industry change with time. For example, in recent years, the number of tourists traveling to South America, the Caribbean, and the Middle East is on the rise. *You will use a system of linear equations to model the trends in tourism in Lesson 7-2.*



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 7.

For Lesson 7-1

Graph Linear Equations

Graph each equation. (For review, see Lesson 4-5.)

1. $y = 1$
2. $y = -2x$
3. $y = 4 - x$
4. $y = 2x + 3$
5. $y = 5 - 2x$
6. $y = \frac{1}{2}x + 2$

For Lesson 7-2

Solve for a Given Variable

Solve each equation or formula for the variable specified. (For review, see Lesson 3-8.)

7. $4x + a = 6x$, for x
8. $8a + y = 16$, for a
9. $\frac{7bc - d}{10} = 12$, for b
10. $\frac{7m + n}{q} = 2m$, for q

For Lessons 7-3 and 7-4

Simplify Expressions

Simplify each expression. If not possible, write *simplified*. (For review, see Lesson 1-5.)

11. $(3x + y) - (2x + y)$
12. $(7x - 2y) - (7x + 4y)$
13. $(16x - 3y) + (11x + 3y)$
14. $(8x - 4y) + (-8x + 5y)$
15. $4(2x + 3y) - (8x - y)$
16. $3(x - 4y) + (x + 12y)$
17. $2(x - 2y) + (3x + 4y)$
18. $5(2x - y) - 2(5x + 3y)$
19. $3(x + 4y) + 2(2x - 6y)$

FOLDABLESTM Study Organizer

Make this Foldable to record information about solving systems of equations and inequalities. Begin with five sheets of grid paper.

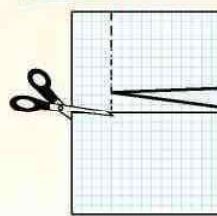
Step 1 Fold

Fold each sheet in half along the width.



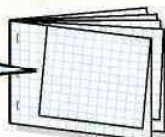
Step 2 Cut

Unfold and cut four rows from left side of each sheet, from the top to the crease.



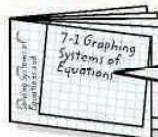
Step 3 Stack and Staple

Stack the sheets and staple to form a booklet.



Step 4 Label

Label each page with a lesson number and title.



Reading and Writing As you read and study the chapter, unfold each page and fill the journal with notes, graphs, and examples for systems of equations and inequalities.



Spreadsheet Investigation

A Preview of Lesson 7-1

Systems of Equations

You can use a spreadsheet to investigate when two quantities will be equal. Enter each formula into the spreadsheet and look for the time when both formulas have the same result.

Example

Bill Winters is considering two job offers in telemarketing departments. The salary at the first job is \$400 per week plus 10% commission on Mr. Winters' sales. At the second job, the salary is \$375 per week plus 15% commission. For what amount of sales would the weekly salary be the same at either job?

Enter different amounts for Mr. Winters' weekly sales in column A. Then enter the formula for the salary at the first job in each cell in column B. In each cell of column C, enter the formula for the salary at the second job.

	A	B	C
1	Sales	Salary 1	Salary 2
2	0	400	375
3	100	410	390
4	200	420	405
5	300	430	420
6	400	440	435
7	500	450	450
8	600	460	465
9	700	470	480
10	800	480	495
11	900	490	510
12	1000	500	525
13			

The spreadsheet shows that for sales of \$500 the total weekly salary for each job is \$450.

Exercises

For Exercises 1–4, use the spreadsheet of weekly salaries above.

1. If x is the amount of Mr. Winters' weekly sales and y is his total weekly salary, write a linear equation for the salary at the first job.
2. Write a linear equation for the salary at the second job.
3. Which ordered pair is a solution for both of the equations you wrote for Exercises 1 and 2?
a. (100, 410) b. (300, 420) c. (500, 450) d. (900, 510)
4. Use the graphing capability of the spreadsheet program to graph the salary data using a line graph. At what point do the two lines intersect? What is the significance of that point in the real-world situation?
5. How could you find the sales for which Mr. Winters' salary will be equal without using a spreadsheet?

7-1

Graphing Systems of Equations

What You'll Learn

- Determine whether a system of linear equations has 0, 1, or infinitely many solutions.
- Solve systems of equations by graphing.

Vocabulary

- system of equations
- consistent
- inconsistent
- independent
- dependent

How can you use graphs to compare the sales of two products?

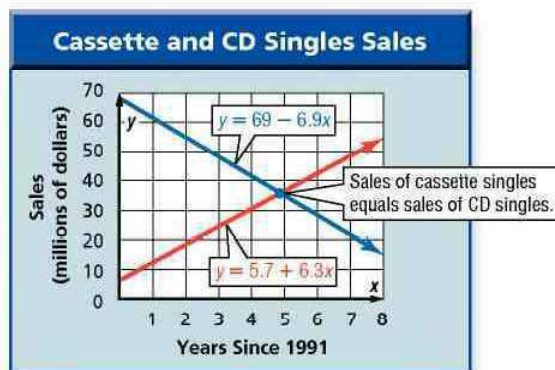
During the 1990s, sales of cassette singles decreased, and sales of CD singles increased. Assume that the sales of these singles were linear functions. If x represents the years since 1991 and y represents the sales in millions of dollars, the following equations represent the sales of these singles.

Cassette singles: $y = 69 - 6.9x$

CD singles: $y = 5.7 + 6.3x$

These equations are graphed at the right.

The point at which the two graphs intersect represents the time when the sales of cassette singles equaled the sales of CD singles. The ordered pair of this point is a solution of both equations.



NUMBER OF SOLUTIONS Two equations, such as $y = 69 - 6.9x$ and $y = 5.7 + 6.3x$, together are called a **system of equations**. A solution of a system of equations is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have 0, 1, or an infinite number of solutions.

- If the graphs intersect or coincide, the system of equations is said to be **consistent**. That is, it has at least one ordered pair that satisfies both equations.
- If the graphs are parallel, the system of equations is said to be **inconsistent**. There are *no* ordered pairs that satisfy both equations.
- Consistent equations can be **independent** or **dependent**. If a system has exactly one solution, it is independent. If the system has an infinite number of solutions, it is dependent.

Concept Summary

Systems of Equations

	Intersecting Lines	Same Line	Parallel Lines
Graph of a System			
Number of Solutions	exactly one solution	infinitely many	no solutions
Terminology	consistent and independent	consistent and dependent	inconsistent

Example 1 Number of Solutions

Use the graph at the right to determine whether each system has *no solution*, *one solution*, or *infinitely many solutions*.

a. $y = -x + 5$
 $y = x - 3$

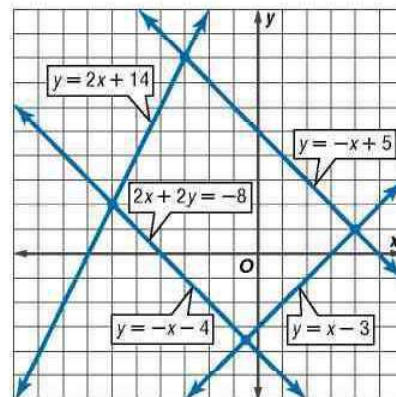
Since the graphs of $y = -x + 5$ and $y = x - 3$ intersect, there is one solution.

b. $y = -x + 5$
 $2x + 2y = -8$

Since the graphs of $y = -x + 5$ and $2x + 2y = -8$ are parallel, there are no solutions.

c. $2x + 2y = -8$
 $y = -x - 4$

Since the graphs of $2x + 2y = -8$ and $y = -x - 4$ coincide, there are infinitely many solutions.



SOLVE BY GRAPHING One method of solving systems of equations is to carefully graph the equations on the same coordinate plane.

Example 2 Solve a System of Equations

Graph each system of equations. Then determine whether the system has *no solution*, *one solution*, or *infinitely many solutions*. If the system has one solution, name it.

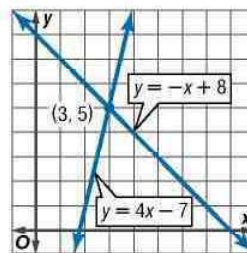
a. $y = -x + 8$
 $y = 4x - 7$

The graphs appear to intersect at the point with coordinates (3, 5). Check this estimate by replacing x with 3 and y with 5 in each equation.

CHECK $y = -x + 8$
 $5 \stackrel{?}{=} -3 + 8$
 $5 = 5 \quad \checkmark$

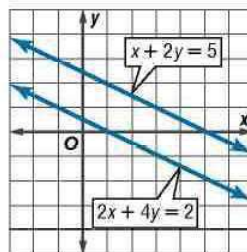
$y = 4x - 7$
 $5 \stackrel{?}{=} 4(3) - 7$
 $5 \stackrel{?}{=} 12 - 7$
 $5 = 5 \quad \checkmark$

The solution is (3, 5).



b. $x + 2y = 5$
 $2x + 4y = 2$

The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions to this system of equations. Notice that the lines have the same slope but different y -intercepts. Recall that a system of equations that has no solution is said to be inconsistent.



Study Tip

Look Back

To review **graphing linear equations**, see Lesson 4-5.



More About...

World Records

In 1994, Guy Delage swam 2400 miles across the Atlantic Ocean from Cape Verde to Barbados. Everyday he would swim awhile and then rest while floating with the current on a huge raft. He averaged 44 miles per day.

Source: Banner Aerospace, Inc.

Example 3 Write and Solve a System of Equations

WORLD RECORDS Use the information on Guy Delage's swim at the left. If Guy can swim 3 miles per hour for an extended period and the raft drifts about 1 mile per hour, how many hours did he spend swimming each day?

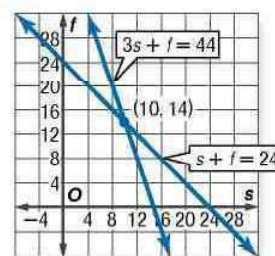
Words You have information about the amount of time spent swimming and floating. You also know the rates and the total distance traveled.

Variables Let s = the number of hours Guy swam, and let f = the number of hours he floated each day. Write a system of equations to represent the situation.

Equations

$\underbrace{\text{The number of hours swimming}}_s$	plus	$\underbrace{\text{the number of hours floating}}_f$	equals	$\underbrace{\text{the total number of hours in a day.}}_{24}$
s	+	f	=	24
$\underbrace{\text{The daily miles traveled swimming}}_{3s}$	plus	$\underbrace{\text{the daily miles traveled floating}}_{1f}$	equals	$\underbrace{\text{the total miles traveled in a day.}}_{44}$
$3s$	+	$1f$	=	44

Graph the equations $s + f = 24$ and $3s + f = 44$. The graphs appear to intersect at the point with coordinates (10, 14). Check this estimate by replacing s with 10 and f with 14 in each equation.



CHECK

$s + f = 24$	$3s + f = 44$
$10 + 14 \stackrel{?}{=} 24$	$3(10) + 14 \stackrel{?}{=} 44$
$24 = 24 \quad \checkmark$	$30 + 14 \stackrel{?}{=} 44$
	$44 = 44 \quad \checkmark$

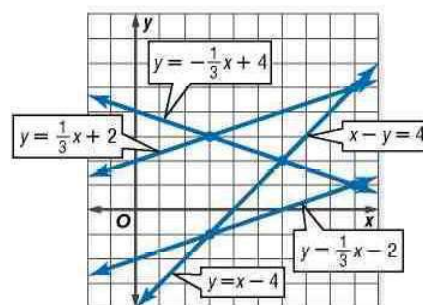
Guy Delage spent about 10 hours swimming each day.

Check for Understanding

- Concept Check**
- OPEN ENDED** Draw the graph of a system of equations that has one solution at $(-2, 3)$.
 - Determine** whether a system of equations with $(0, 0)$ and $(2, 2)$ as solutions *sometimes*, *always*, or *never* has other solutions. Explain.
 - Find a counterexample** for the following statement.
If the graphs of two linear equations have the same slope, then the system of equations has no solution.

Guided Practice Use the graph at the right to determine whether each system has *no solution*, *one solution*, or *infinitely many solutions*.

- | | |
|------------------------|---------------------------|
| 4. $y = x - 4$ | 5. $y = \frac{1}{3}x + 2$ |
| $y = \frac{1}{3}x - 2$ | $y = \frac{1}{3}x - 2$ |
| 6. $x - y = 4$ | 7. $x - y = 4$ |
| $y = x - 4$ | $y = -\frac{1}{3}x + 4$ |



Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

- | | | |
|--------------------------------|----------------------------------|------------------------------------|
| 8. $y = -x$
$y = 2x$ | 9. $x + y = 8$
$x - y = 2$ | 10. $2x + 4y = 2$
$3x + 6y = 3$ |
| 11. $x + y = 4$
$x + y = 1$ | 12. $x - y = 2$
$3y + 2x = 9$ | 13. $x + y = 2$
$y = 4x + 7$ |

Application 14. **RESTAURANTS** The Rodriguez family and the Wong family went to a brunch buffet. The restaurant charges one price for adults and another price for children. The Rodriguez family has two adults and three children, and their bill was \$40.50. The Wong family has three adults and one child, and their bill was \$38.00. Determine the price of the buffet for an adult and the price for a child.

Practice and Apply

Homework Help

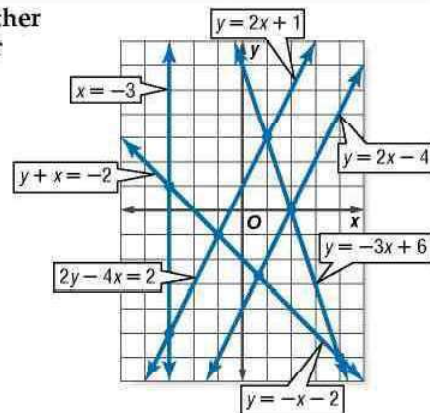
For Exercises	See Examples
15–22	1
23–40	2
41–54	3

Extra Practice

See page 835.

Use the graph at the right to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.

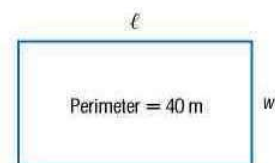
- | | |
|------------------------------------|-----------------------------------|
| 15. $x = -3$
$y = 2x + 1$ | 16. $y = -x - 2$
$y = 2x - 4$ |
| 17. $y + x = -2$
$y = -x - 2$ | 18. $y = 2x + 1$
$y = 2x - 4$ |
| 19. $y = -3x + 6$
$y = 2x - 4$ | 20. $2y - 4x = 2$
$y = 2x - 4$ |
| 21. $2y - 4x = 2$
$y = -3x + 6$ | 22. $2y - 4x = 2$
$y = 2x + 1$ |



Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

- | | | |
|---|---|---|
| 23. $y = -6$
$4x + y = 2$ | 24. $x = 2$
$3x - y = 8$ | 25. $y = \frac{1}{2}x$
$2x + y = 10$ |
| 26. $y = -x$
$y = 2x - 6$ | 27. $y = 3x - 4$
$y = -3x - 4$ | 28. $y = 2x + 6$
$y = -x - 3$ |
| 29. $x - 2y = 2$
$3x + y = 6$ | 30. $x + y = 2$
$2y - x = 10$ | 31. $3x + 2y = 12$
$3x + 2y = 6$ |
| 32. $2x + 3y = 4$
$-4x - 6y = -8$ | 33. $2x + y = -4$
$5x + 3y = -6$ | 34. $4x + 3y = 24$
$5x - 8y = -17$ |
| 35. $3x + y = 3$
$2y = -6x + 6$ | 36. $y = x + 3$
$3y + x = 5$ | 37. $2x + 3y = -17$
$y = x - 4$ |
| 38. $y = \frac{2}{3}x - 5$
$3y = 2x$ | 39. $6 - \frac{3}{8}y = x$
$\frac{2}{3}x + \frac{1}{4}y = 4$ | 40. $\frac{1}{2}x + \frac{1}{3}y = 6$
$y = \frac{1}{2}x + 2$ |

41. **GEOMETRY** The length of the rectangle at the right is 1 meter less than twice its width. What are the dimensions of the rectangle?



WebQuest

You can graph a system of equations to predict when men's and women's Olympic times will be the same. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

GEOMETRY For Exercises 42 and 43, use the graphs of $y = 2x + 6$, $3x + 2y = 19$, and $y = 2$, which contain the sides of a triangle.

42. Find the coordinates of the vertices of the triangle.
43. Find the area of the triangle.

BALLOONING For Exercises 44 and 45, use the information in the graphic at the right.

44. In how many minutes will the balloons be at the same height?
45. How high will the balloons be at that time?

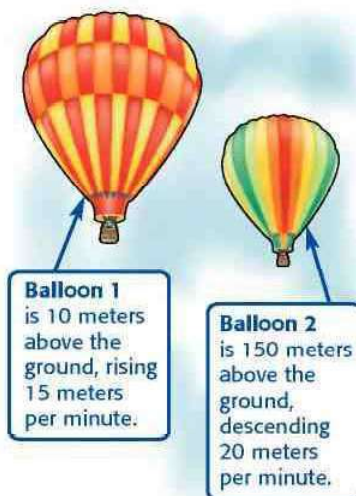
SAVINGS For Exercises 46 and 47, use the following information.

Monica and Michael Gordon both want to buy a scooter. Monica has already saved \$25 and plans to save \$5 per week until she can buy the scooter. Michael has \$16 and plans to save \$8 per week.

46. In how many weeks will Monica and Michael have saved the same amount of money?
47. How much will each person have saved at that time?

BUSINESS For Exercises 48–50, use the graph at the right.

48. Which company had the greater profit during the ten years?
49. Which company had a greater rate of growth?
50. If the profit patterns continue, will the profits of the two companies ever be equal? Explain.



POPULATION For Exercises 51–54, use the following information.

The U.S. Census Bureau divides the country into four sections. They are the Northeast, the Midwest, the South, and the West.

51. In 1990, the population of the Midwest was about 60 million. During the 1990s, the population of this area increased an average of about 0.4 million per year. Write an equation to represent the population of the Midwest for the years since 1990.
52. The population of the West was about 53 million in 1990. The population of this area increased an average of about 1 million per year during the 1990s. Write an equation to represent the population of the West for the years since 1990.
53. Graph the population equations.
54. Assume that the rate of growth of each of these areas remains the same. Estimate when the population of the West would be equal to the population of the Midwest.
55. **CRITICAL THINKING** The solution of the system of equations $Ax + y = 5$ and $Ax + By = 20$ is $(2, -3)$. What are the values of A and B ?



56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

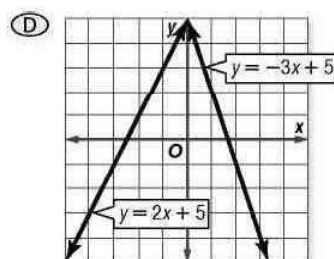
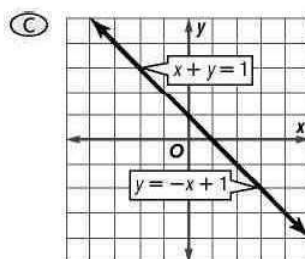
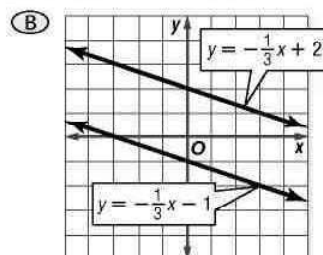
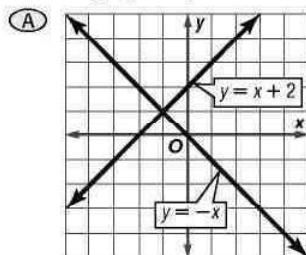
How can you use graphs to compare the sales of two products?

Include the following in your answer:

- an estimate of the year in which the sales of cassette singles equaled the sales of CD singles, and
- an explanation of why graphing works.

**Standardized
Test Practice**

57. Which graph represents a system of equations with no solution?



58. How many solutions exist for the system of equations below?

$$4x + y = 7$$

$$3x - y = 0$$

- (A) no solution (B) one solution
(C) infinitely many solutions (D) cannot be determined

Maintain Your Skills

Mixed Review

Determine which ordered pairs are part of the solution set for each inequality.

(Lesson 6-6)

59. $y \leq 2x$, $\{(1, 4), (-1, 5), (5, -6), (-7, 0)\}$

60. $y < 8 - 3x$, $\{(-4, 2), (-3, 0), (1, 4), (1, 8)\}$

61. **MANUFACTURING** The inspector at a perfume manufacturer accepts a bottle if it is less than 0.05 ounce above or below 2 ounces. What are the acceptable numbers of ounces for a perfume bottle? (Lesson 6-5)

Write each equation in standard form. (Lesson 5-5)

62. $y - 1 = 4(x - 5)$

63. $y + 2 = \frac{1}{3}(x + 3)$

64. $y - 4 = -6(x + 2)$

**Getting Ready for
the Next Lesson**

PREREQUISITE SKILL Solve each equation for the variable specified.

(To review solving equations for a specified variable, see Lesson 3-8.)

65. $12x - y = 10x$, for y

66. $6a + b = 2a$, for a

67. $\frac{7m - n}{q} = 10$, for q

68. $\frac{5tz - s}{2} = 6$, for z



Graphing Calculator Investigation

A Follow-Up of Lesson 7-1

Systems of Equations

You can use a TI-83 Plus graphing calculator to solve a system of equations.

Example

Solve the system of equations. State the decimal solution to the nearest hundredth.

$$2.93x + y = 6.08$$

$$8.32x - y = 4.11$$

Step 1 Solve each equation for y to enter them into the calculator.

$$2.93x + y = 6.08$$

First equation

$$2.93x + y - 2.93x = 6.08 - 2.93x$$

Subtract $2.93x$ from each side.

$$y = 6.08 - 2.93x$$

Simplify.

$$8.32x - y = 4.11$$

Second equation

$$8.32x - y - 8.32x = 4.11 - 8.32x$$

Subtract $8.32x$ from each side.

$$-y = 4.11 - 8.32x$$

Simplify.

$$(-1) \cdot y = (-1)(4.11 - 8.32x)$$

Multiply each side by -1 .

$$y = -4.11 + 8.32x$$

Simplify.

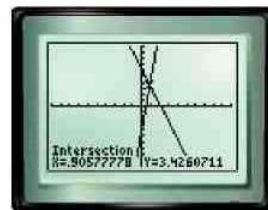
Step 2 Enter these equations in the $Y=$ list and graph.

KEYSTROKES: Review on pages 224–225.

Step 3 Use the **CALC** menu to find the point of intersection.

KEYSTROKES: **2nd** **[CALC]** **5** **ENTER** **ENTER** **ENTER**

The solution is approximately $(0.91, 3.43)$.



$[10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

Exercises

Use a graphing calculator to solve each system of equations. Write decimal solutions to the nearest hundredth.

1. $y = 3x - 4$

$$y = -0.5x + 6$$

3. $x + y = 5.35$

$$3x - y = 3.75$$

5. $1.5x + y = 6.7$

$$5.2x - y = 4.1$$

7. $5x - 4y = 26$

$$4x + 2y = 53.3$$

9. $0.22x + 0.15y = 0.30$

$$-0.33x + y = 6.22$$

2. $y = 2x + 5$

$$y = -0.2x - 4$$

4. $0.35x - y = 1.12$

$$2.25x + y = -4.05$$

6. $5.4x - y = 1.8$

$$6.2x + y = -3.8$$

8. $2x + 3y = 11$

$$4x + y = -6$$

10. $125x - 200y = 800$

$$65x - 20y = 140$$



www.algebra1.com/other_calculator_keystrokes

7-2 Substitution

What You'll Learn

- Solve systems of equations by using substitution.
- Solve real-world problems involving systems of equations.

Vocabulary

- substitution

How can a system of equations be used to predict media use?

Americans spend more time online than they spend reading daily newspapers. If x represents the number of years since 1993 and y represents the average number of hours per person per year, the following system represents the situation.

reading daily newspapers: $y = -2.8x + 170$
 online: $y = 14.4x + 2$

The solution of the system represents the year that the number of hours spent on each activity will be the same. To solve this system, you could graph the equations and find the point of intersection. However, the exact coordinates of the point would be very difficult to determine from the graph. You could find a more accurate solution by using algebraic methods.



SUBSTITUTION The exact solution of a system of equations can be found by using algebraic methods. One such method is called **substitution**.



Algebra Activity

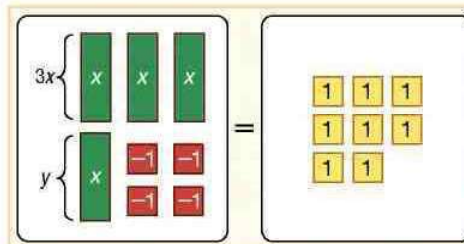
Using Substitution

Use algebra tiles and an equation mat to solve the system of equations.
 $3x + y = 8$ and $y = x - 4$

Model and Analyze

Since $y = x - 4$, use 1 positive x tile and 4 negative 1 tiles to represent y . Use algebra tiles to represent $3x + y = 8$.

- Use what you know about equation mats to solve for x . What is the value of x ?
- Use the $y = x - 4$ to solve for y .
- What is the solution of the system of equations?



Make a Conjecture

- Explain how to solve the following system of equations using algebra tiles.
 $4x + 3y = 10$ and $y = x + 1$
- Why do you think this method is called substitution?

Study Tip**Look Back**

To review **solving linear equations**, see Lesson 3-5.

Example 1 Solve Using Substitution

Use substitution to solve the system of equations.

$$\begin{aligned} y &= 3x \\ x + 2y &= -21 \end{aligned}$$

Since $y = 3x$, substitute $3x$ for y in the second equation.

$$\begin{aligned} x + 2y &= -21 && \text{Second equation} \\ x + 2(3x) &= -21 && y = 3x \\ x + 6x &= -21 && \text{Simplify.} \\ 7x &= -21 && \text{Combine like terms.} \\ \frac{7x}{7} &= \frac{-21}{7} && \text{Divide each side by 7.} \\ x &= -3 && \text{Simplify.} \end{aligned}$$

Use $y = 3x$ to find the value of y .

$$\begin{aligned} y &= 3x && \text{First equation} \\ y &= 3(-3) && x = -3 \\ y &= -9 && \text{The solution is } (-3, -9). \end{aligned}$$

Example 2 Solve for One Variable, Then Substitute

Use substitution to solve the system of equations.

$$\begin{aligned} x + 5y &= -3 \\ 3x - 2y &= 8 \end{aligned}$$

Solve the first equation for x since the coefficient of x is 1.

$$\begin{aligned} x + 5y &= -3 && \text{First equation} \\ x + 5y - 5y &= -3 - 5y && \text{Subtract } 5y \text{ from each side.} \\ x &= -3 - 5y && \text{Simplify.} \end{aligned}$$

Find the value of y by substituting $-3 - 5y$ for x in the second equation.

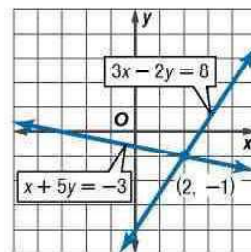
$$\begin{aligned} 3x - 2y &= 8 && \text{Second equation} \\ 3(-3 - 5y) - 2y &= 8 && x = -3 - 5y \\ -9 - 15y - 2y &= 8 && \text{Distributive Property} \\ -9 - 17y &= 8 && \text{Combine like terms.} \\ -9 - 17y + 9 &= 8 + 9 && \text{Add 9 to each side.} \\ -17y &= 17 && \text{Simplify.} \\ \frac{-17y}{-17} &= \frac{17}{-17} && \text{Divide each side by } -17. \\ y &= -1 && \text{Simplify.} \end{aligned}$$

Substitute -1 for y in either equation to find the value of x .

Choose the equation that is easier to solve.

$$\begin{aligned} x + 5y &= -3 && \text{First equation} \\ x + 5(-1) &= -3 && y = -1 \\ x - 5 &= -3 && \text{Simplify.} \\ x &= 2 && \text{Add 5 to each side.} \end{aligned}$$

The solution is $(2, -1)$. The graph verifies the solution.



Example 3 Dependent System

Use substitution to solve the system of equations.

$$6x - 2y = -4$$

$$y = 3x + 2$$

Since $y = 3x + 2$, substitute $3x + 2$ for y in the first equation.

$$6x - 2y = -4 \quad \text{First equation}$$

$$6x - 2(3x + 2) = -4 \quad y = 3x + 2$$

$$6x - 6x - 4 = -4 \quad \text{Distributive Property}$$

$$-4 = -4 \quad \text{Simplify.}$$

The statement $-4 = -4$ is true. This means that there are infinitely many solutions of the system of equations. This is true because the slope-intercept form of both equations is $y = 3x + 2$. That is, the equations are equivalent, and they have the same graph.

In general, if you solve a system of linear equations and the result is a true statement (an identity such as $-4 = -4$), the system has an infinite number of solutions. However, if the result is a false statement (for example, $-4 = 5$), the system has no solution.

Study Tip

Alternative Method

Using a system of equations is an alternative method for solving the weighted average problems that you studied in Lesson 3-9.

REAL-WORLD PROBLEMS Sometimes it is helpful to organize data before solving a problem. Some ways to organize data are to use tables, charts, different types of graphs, or diagrams.

Example 4 Write and Solve a System of Equations

METAL ALLOYS A metal alloy is 25% copper. Another metal alloy is 50% copper. How much of each alloy should be used to make 1000 grams of a metal alloy that is 45% copper?

Let a = the number of grams of the 25% copper alloy and b = the number of grams of the 50% copper alloy. Use a table to organize the information.

	25% Copper	50% Copper	45% Copper
Total Grams	a	b	1000
Grams of Copper	$0.25a$	$0.50b$	$0.45(1000)$

The system of equations is $a + b = 1000$ and $0.25a + 0.50b = 0.45(1000)$. Use substitution to solve this system.

$$a + b = 1000 \quad \text{First equation}$$

$$a + b - b = 1000 - b \quad \text{Subtract } b \text{ from each side.}$$

$$a = 1000 - b \quad \text{Simplify.}$$

$$0.25a + 0.50b = 0.45(1000) \quad \text{Second equation}$$

$$0.25(1000 - b) + 0.50b = 0.45(1000) \quad a = 1000 - b$$

$$250 - 0.25b + 0.50b = 450 \quad \text{Distributive Property}$$

$$250 + 0.25b = 450 \quad \text{Combine like terms.}$$

$$250 + 0.25b - 250 = 450 - 250 \quad \text{Subtract 250 from each side.}$$

$$0.25b = 200 \quad \text{Simplify.}$$

$$\frac{0.25b}{0.25} = \frac{200}{0.25} \quad \text{Divide each side by 0.25.}$$

$$b = 800 \quad \text{Simplify.}$$

$$\begin{array}{ll}
 a + b = 1000 & \text{First equation} \\
 a + 800 = 1000 & b = 800 \\
 a + 800 - 800 = 1000 - 800 & \text{Subtract 800 from each side.} \\
 a = 200 & \text{Simplify.}
 \end{array}$$

200 grams of the 25% copper alloy and 800 grams of the 50% copper alloy should be used.

Check for Understanding

Concept Check

1. Explain why you might choose to use substitution rather than graphing to solve a system of equations.
2. Describe the graphs of two equations if the solution of the system of equations yields the equation $4 = 2$.
3. **OPEN-ENDED** Write a system of equations that has infinitely many solutions.

Guided Practice

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

- | | | |
|-------------------|------------------|-----------------------|
| 4. $x = 2y$ | 5. $y = 3x - 8$ | 6. $2x + 7y = 3$ |
| $4x + 2y = 15$ | $y = 4 - x$ | $x = 1 - 4y$ |
| 7. $6x - 2y = -4$ | 8. $x + 3y = 12$ | 9. $y = \frac{3}{5}x$ |
| $y = 3x + 2$ | $x - y = 8$ | $3x - 5y = 15$ |

Application

10. **TRANSPORTATION** The Thrust SSC is the world's fastest land vehicle. Suppose the driver of a car whose top speed is 200 miles per hour requests a race against the SSC. The car gets a head start of one-half hour. If there is unlimited space to race, at what distance will the SSC pass the car?



Practice and Apply

Homework Help

For Exercises	See Examples
11–28	1–3
29–37	4

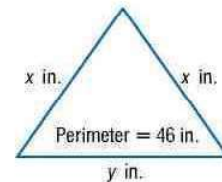
Extra Practice

See page 835.

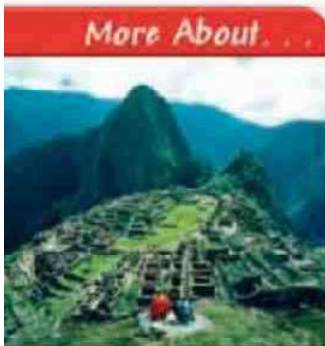
Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

- | | | |
|----------------------|----------------------------|----------------------------|
| 11. $y = 5x$ | 12. $x = 4y$ | 13. $x = 4y + 5$ |
| $2x + 3y = 34$ | $2x + 3y = 44$ | $x = 3y - 2$ |
| 14. $y = 2x + 3$ | 15. $4c = 3d + 3$ | 16. $4x + 5y = 11$ |
| $y = 4x - 1$ | $c = d - 1$ | $y = 3x - 13$ |
| 17. $8x + 2y = 13$ | 18. $2x - y = -4$ | 19. $3x - 5y = 11$ |
| $4x + y = 11$ | $-3x + y = -9$ | $x - 3y = 1$ |
| 20. $2x + 3y = 1$ | 21. $c - 5d = 2$ | 22. $5r - s = 5$ |
| $-3x + y = 15$ | $2c + d = 4$ | $-4r + 5s = 17$ |
| 23. $3x - 2y = 12$ | 24. $x - 3y = 0$ | 25. $-0.3x + y = 0.5$ |
| $x + 2y = 6$ | $3x + y = 7$ | $0.5x - 0.3y = 1.9$ |
| 26. $0.5x - 2y = 17$ | 27. $y = \frac{1}{2}x + 3$ | 28. $x = \frac{1}{2}y + 3$ |
| $2x + y = 104$ | $y = 2x - 1$ | $2x - y = 6$ |

29. **GEOMETRY** The base of the triangle is 4 inches longer than the length of one of the other sides. Use a system of equations to find the length of each side of the triangle.



30. **FUND-RAISING** The Future Teachers of America Club at Paint Branch High School is making a healthy trail mix to sell to students during lunch. The mix will have three times the number of pounds of raisins as sunflower seeds. Sunflower seeds cost \$4.00 per pound, and raisins cost \$1.50 per pound. If the group has \$34.00 to spend on the raisins and sunflower seeds, how many pounds of each should they buy?
31. **CHEMISTRY** MX Labs needs to make 500 gallons of a 34% acid solution. The only solutions available are a 25% acid solution and a 50% acid solution. How many gallons of each solution should be mixed to make the 34% solution?
32. **GEOMETRY** Supplementary angles are two angles whose measures have the sum of 180 degrees. Angles X and Y are supplementary, and the measure of angle X is 24 degrees greater than the measure of angle Y. Find the measures of angles X and Y.
33. **SPORTS** At the end of the 2000 baseball season, the New York Yankees and the Cincinnati Reds had won a total of 31 World Series. The Yankees had won 5.2 times as many World Series as the Reds. How many World Series did each team win?



Tourism

Every year, multitudes of visitors make their way to South America to stand in awe of Machu Picchu, the spectacular ruins of the Lost City of the Incas.

Source: www.about.com

JOBS For Exercises 34 and 35, use the following information.

Shantel Jones has two job offers as a car salesperson. At one dealership, she will receive \$600 per month plus a commission of 2% of the total price of the automobiles she sells. At the other dealership, she will receive \$1000 per month plus a commission of 1.5% of her total sales.

34. What is the total price of the automobiles that Ms. Jones must sell each month to make the same income from either dealership?
35. Explain which job offer is better.

36. **LANDSCAPING** A blue spruce grows an average of 6 inches per year. A hemlock grows an average of 4 inches per year. If a blue spruce is 4 feet tall and a hemlock is 6 feet tall, when would you expect the trees to be the same height?

37. **TOURISM** In 2000, approximately 40.3 million tourists visited South America and the Caribbean. The number of tourists to that area had been increasing at an average rate of 0.8 million tourists per year. In the same year, 17.0 million tourists visited the Middle East. The number of tourists to the Middle East had been increasing at an average rate of 1.8 million tourists per year. If the trend continues, when would you expect the number of tourists to South America and the Caribbean to equal the number of tourists to the Middle East?

38. **RESEARCH** Use the Internet or other resources to find the pricing plans for various cell phones. Determine the number of minutes you would need to use the phone for two plans to cost the same amount of money. Support your answer with a table, a graph, and/or an equation.

39. **CRITICAL THINKING** Solve the system of equations. Write the solution as an ordered triple of the form (x, y, z) .

$$\begin{aligned}2x + 3y - z &= 17 \\ y &= -3z - 7 \\ 2x &= z + 2\end{aligned}$$

40. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a system of equations be used to predict media use?

Include the following in your answer:

- an explanation of solving a system of equations by using substitution, and
- the year when the number of hours spent reading daily newspapers is the same as the hours spent online.



41. When solving the following system, which expression could be substituted for x ?

$$\begin{aligned}x + 4y &= 1 \\ 2x - 3y &= -9\end{aligned}$$

- (A) $4y - 1$ (B) $1 - 4y$ (C) $3y - 9$ (D) $-9 - 3y$

42. If $x - 3y = -9$ and $5x - 2y = 7$, what is the value of x ?

- (A) 1 (B) 2 (C) 3 (D) 4

Maintain Your Skills

Mixed Review

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. (Lesson 7-1)

43. $\begin{aligned}x + y &= 3 \\ x + y &= 4\end{aligned}$

44. $\begin{aligned}x + 2y &= 1 \\ 2x + y &= 5\end{aligned}$

45. $\begin{aligned}2x + y &= 3 \\ 4x + 2y &= 6\end{aligned}$

Graph each inequality. (Lesson 6-6)

46. $y < -5$

47. $x \geq 4$

48. $2x + y > 6$

49. **RECYCLING** When a pair of blue jeans is made, the leftover denim scraps can be recycled. One pound of denim is left after making every five pair of jeans. How many pounds of denim would be left from 250 pairs of jeans? (Lesson 3-6)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Simplify each expression.

(To review *simplifying expressions*, see Lesson 1-5.)

50. $6a - 9a$

51. $8t + 4t$

52. $-7g - 8g$

53. $7d - (2d + b)$

Practice Quiz 1

Lessons 7-1 and 7-2

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. (Lesson 7-1)

1. $\begin{aligned}x + y &= 3 \\ x - y &= 1\end{aligned}$

2. $\begin{aligned}3x - 2y &= -6 \\ 3x - 2y &= 6\end{aligned}$

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions. (Lesson 7-2)

3. $\begin{aligned}x + y &= 0 \\ 3x + y &= -8\end{aligned}$

4. $\begin{aligned}x - 2y &= 5 \\ 3x - 5y &= 8\end{aligned}$

5. $\begin{aligned}x + y &= 2 \\ y &= 2 - x\end{aligned}$



7-3

Elimination Using Addition and Subtraction

What You'll Learn

- Solve systems of equations by using elimination with addition.
- Solve systems of equations by using elimination with subtraction.

Vocabulary

- elimination

How can you use a system of equations to solve problems about weather?

On the winter solstice, there are fewer hours of daylight in the Northern Hemisphere than on any other day. On that day in Seward, Alaska, the difference between the number of hours of darkness n and the number of hours of daylight d is 12. The following system of equations represents the situation.

$$\begin{aligned}n + d &= 24 \\n - d &= 12\end{aligned}$$

Notice that if you add these equations, the variable d is eliminated.

$$\begin{array}{r}n + d = 24 \\(+)\ n - d = 12 \\ \hline 2n = 36\end{array}$$

ELIMINATION USING ADDITION Sometimes adding two equations together will eliminate one variable. Using this step to solve a system of equations is called **elimination**.

Example 1 Elimination Using Addition

Use elimination to solve each system of equations.

$$\begin{aligned}3x - 5y &= -16 \\2x + 5y &= 31\end{aligned}$$

Since the coefficients of the y terms, -5 and 5 , are additive inverses, you can eliminate the y terms by adding the equations.

$$\begin{array}{r}3x - 5y = -16 \\(+)\ 2x + 5y = 31 \\ \hline 5x = 15 \\ \frac{5x}{5} = \frac{15}{5} \\ x = 3\end{array}$$

Write the equations in column form and add.

Notice that the y variable is eliminated.

Divide each side by 5.

Simplify.

Now substitute 3 for x in either equation to find the value of y .

$$\begin{aligned}3x - 5y &= -16 && \text{First equation} \\3(3) - 5y &= -16 && \text{Replace } x \text{ with } 3. \\9 - 5y &= -16 && \text{Simplify.} \\9 - 5y - 9 &= -16 - 9 && \text{Subtract 9 from each side.} \\-5y &= -25 && \text{Simplify.} \\ \frac{-5y}{-5} &= \frac{-25}{-5} && \text{Divide each side by } -5. \\ y &= 5 && \text{Simplify.}\end{aligned}$$

The solution is $(3, 5)$.

Study Tip

Look Back

To review translating verbal sentences into equations, see Lesson 3-1.

Example 2 Write and Solve a System of Equations

Twice one number added to another number is 18. Four times the first number minus the other number is 12. Find the numbers.

Let x represent the first number and y represent the second number.

$$\begin{array}{rclclcl} \text{Twice one number} & & \text{added to} & & \text{another number} & \text{is} & 18 \\ 2x & & + & & y & = & 18 \\ \hline \text{Four times the first number} & & \text{minus} & & \text{the other number} & \text{is} & 12 \\ 4x & & - & & y & = & 12 \end{array}$$

Use elimination to solve the system.

$$\begin{array}{r} 2x + y = 18 \\ (+) 4x - y = 12 \\ \hline 6x = 30 \\ 6x = 30 \\ \hline x = 5 \end{array}$$

Write the equations in column form and add.

Notice that the variable y is eliminated.

Divide each side by 6.

Simplify.

Now substitute 5 for x in either equation to find the value of y .

$$\begin{array}{r} 4x - y = 12 \\ 4(5) - y = 12 \\ 20 - y = 12 \\ 20 - y = 20 = 12 - 20 \\ -y = -8 \\ \frac{-y}{-1} = \frac{-8}{-1} \\ y = 8 \end{array}$$

Second equation

Replace x with 5.

Simplify.

Subtract 20 from each side.

Simplify.

Divide each side by -1 .

The numbers are 5 and 8.

ELIMINATION USING SUBTRACTION Sometimes subtracting one equation from another will eliminate one variable.

Example 3 Elimination Using Subtraction

Use elimination to solve the system of equations.

$$\begin{array}{r} 5s + 2t = 6 \\ 9s + 2t = 22 \end{array}$$

Since the coefficients of the t terms, 2 and 2, are the same, you can eliminate the t terms by subtracting the equations.

$$\begin{array}{r} 5s + 2t = 6 \\ (-) 9s + 2t = 22 \\ \hline -4s = -16 \\ \frac{-4s}{-4} = \frac{-16}{-4} \\ s = 4 \end{array}$$

Write the equations in column form and subtract.

Notice that the variable t is eliminated.

Divide each side by -4 .

Simplify.

Now substitute 4 for s in either equation to find the value of t .

$$\begin{array}{r} 5s + 2t = 6 \\ 5(4) + 2t = 6 \\ 20 + 2t = 6 \\ 20 + 2t = 20 = 6 - 20 \\ 2t = -14 \\ \frac{2t}{2} = \frac{-14}{2} \\ t = -7 \end{array}$$

First equation

$s = 4$

Simplify.

Subtract 20 from each side.

Simplify.

Divide each side by 2.

The solution is $(4, -7)$.



Example 4 Elimination Using Subtraction

Multiple-Choice Test Item

If $x - 3y = 7$ and $x + 2y = 2$, what is the value of x ?

- (A) 4 (B) -1 (C) (-1, 4) (D) (4, -1)



Test-Taking Tip

Always read the question carefully. Ask yourself, "What does the question ask?" Then answer that question.

Read the Test Item

You are given a system of equations, and you are asked to find the value of x .

Solve the Test Item

You can eliminate the x terms by subtracting one equation from the other.

$$\begin{array}{r} x - 3y = 7 \\ (-) x + 2y = 2 \\ \hline \end{array}$$

$-5y = 5$ Notice the x variable is eliminated.

$$\frac{-5y}{-5} = \frac{5}{-5}$$

$y = -1$ Simplify.

Now substitute -1 for y in either equation to find the value of x .

$$x + 2y = 2 \quad \text{Second equation}$$

$$x + 2(-1) = 2 \quad y = -1$$

$$x - 2 = 2 \quad \text{Simplify.}$$

$$x - 2 + 2 = 2 + 2 \quad \text{Add 2 to each side.}$$

$$x = 4 \quad \text{Simplify.}$$

Notice that B is the value of y and D is the solution of the system of equations. However, the question asks for the value of x . The answer is A.

Check for Understanding

Concept Check

- OPEN ENDED** Write a system of equations that can be solved by using addition to eliminate one variable.
- Describe** a system of equations that can be solved by using subtraction to eliminate one variable.
- FIND THE ERROR** Michael and Yoomee are solving a system of equations.

Michael

$$\begin{array}{r} 2r + s = 5 \\ (+) r - s = 1 \\ \hline 3r = 6 \\ r = 2 \\ 2r + s = 5 \\ 2(2) + s = 5 \\ 4 + s = 5 \\ s = 1 \\ \text{The solution is } (2, 1). \end{array}$$

Yoomee

$$\begin{array}{r} 2r + s = 5 \\ (-) r - s = 1 \\ \hline r = 4 \\ r - s = 1 \\ 4 - s = 1 \\ -s = -3 \\ s = 3 \\ \text{The solution is } (4, 3). \end{array}$$

Who is correct? Explain your reasoning.

Guided Practice

Use elimination to solve each system of equations.

$$\begin{aligned} 4. \quad x - y &= 14 \\ x + y &= 20 \end{aligned}$$

$$\begin{aligned} 5. \quad 2a - 3b &= -11 \\ a + 3b &= 8 \end{aligned}$$

$$\begin{aligned} 6. \quad 4x + y &= -9 \\ 4x + 2y &= -10 \end{aligned}$$

$$\begin{aligned} 7. \quad 6x + 2y &= -10 \\ 2x + 2y &= -10 \end{aligned}$$

$$\begin{aligned} 8. \quad 2a + 4b &= 30 \\ -2a - 2b &= -21.5 \end{aligned}$$

$$\begin{aligned} 9. \quad -4m + 2n &= 6 \\ -4m + n &= 8 \end{aligned}$$

10. The sum of two numbers is 24. Five times the first number minus the second number is 12. What are the two numbers?

Standardized Test Practice

11. If $2x + 7y = 17$ and $2x + 5y = 11$, what is the value of $2y$?

(A) -4

(B) -2

(C) 3

(D) 6

Practice and Apply

Homework Help

For Exercises	See Examples
12–29	1, 3
30–39	2
42, 43	4

Extra Practice

See page 836.

Use elimination to solve each system of equations.

$$\begin{aligned} 12. \quad x + y &= -3 \\ x - y &= 1 \end{aligned}$$

$$\begin{aligned} 13. \quad s - t &= 4 \\ s + t &= 2 \end{aligned}$$

$$\begin{aligned} 14. \quad 3m - 2n &= 13 \\ m + 2n &= 7 \end{aligned}$$

$$\begin{aligned} 15. \quad -4x + 2y &= 8 \\ 4x - 3y &= -10 \end{aligned}$$

$$\begin{aligned} 16. \quad 3a + b &= 5 \\ 2a + b &= 10 \end{aligned}$$

$$\begin{aligned} 17. \quad 2m - 5n &= -6 \\ 2m - 7n &= -14 \end{aligned}$$

$$\begin{aligned} 18. \quad 3r - 5s &= -35 \\ 2r - 5s &= -30 \end{aligned}$$

$$\begin{aligned} 19. \quad 13a + 5b &= -11 \\ 13a + 11b &= 7 \end{aligned}$$

$$\begin{aligned} 20. \quad 3x - 5y &= 16 \\ -3x + 2y &= -10 \end{aligned}$$

$$\begin{aligned} 21. \quad 6s + 5t &= 1 \\ 6s - 5t &= 11 \end{aligned}$$

$$\begin{aligned} 22. \quad 4x - 3y &= 12 \\ 4x + 3y &= 24 \end{aligned}$$

$$\begin{aligned} 23. \quad a - 2b &= 5 \\ 3a - 2b &= 9 \end{aligned}$$

$$\begin{aligned} 24. \quad 4x + 5y &= 7 \\ 8x + 5y &= 9 \end{aligned}$$

$$\begin{aligned} 25. \quad 8a + b &= 1 \\ 8a - 3b &= 3 \end{aligned}$$

$$\begin{aligned} 26. \quad 1.44x - 3.24y &= -5.58 \\ 1.08x + 3.24y &= 9.99 \end{aligned}$$

$$\begin{aligned} 27. \quad 7.2m + 4.5n &= 129.06 \\ 7.2m + 6.7n &= 136.54 \end{aligned}$$

$$\begin{aligned} 28. \quad \frac{3}{5}c - \frac{1}{5}d &= 9 \\ \frac{7}{5}c + \frac{1}{5}d &= 11 \end{aligned}$$

$$\begin{aligned} 29. \quad \frac{2}{3}x - \frac{1}{2}y &= 14 \\ \frac{5}{6}x - \frac{1}{2}y &= 18 \end{aligned}$$

30. The sum of two numbers is 48, and their difference is 24. What are the numbers?

31. Find the two numbers whose sum is 51 and whose difference is 13.

32. Three times one number added to another number is 18. Twice the first number minus the other number is 12. Find the numbers.

33. One number added to twice another number is 23. Four times the first number added to twice the other number is 38. What are the numbers?

34. **BUSINESS** In 1999, the United States produced about 2 million more motor vehicles than Japan. Together, the two countries produced about 22 million motor vehicles. How many vehicles were produced in each country?

35. **PARKS** A youth group and their leaders visited Mammoth Cave. Two adults and 5 students in one van paid \$77 for the Grand Avenue Tour of the cave. Two adults and 7 students in a second van paid \$95 for the same tour. Find the adult price and the student price of the tour.

36. **FOOTBALL** During the National Football League's 1999 season, Troy Aikman, the quarterback for the Dallas Cowboys, earned \$0.467 million more than Deion Sanders, the Cowboys cornerback. Together they cost the Cowboys \$12.867 million. How much did each player make?

More About...



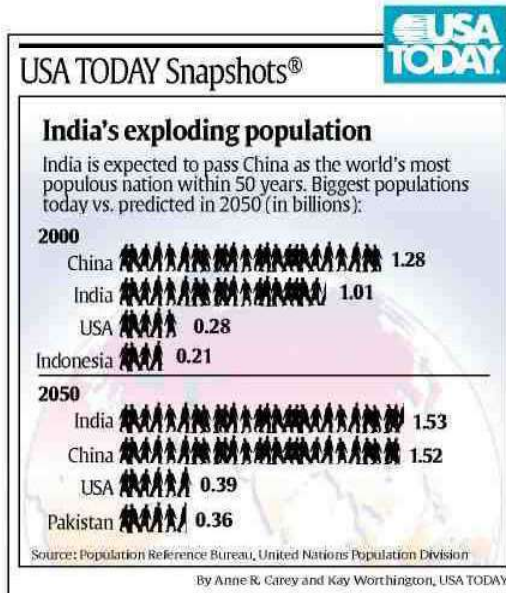
Parks

Mammoth Cave in Kentucky was declared a national park in 1941. It has more than 336 miles of explored caves, making it the longest recorded cave system in the world.

Source: National Park Service

POPULATIONS For Exercises 37–39, use the information in the graph at the right.

37. Let x represent the number of years since 2000 and y represent population in billions. Write an equation to represent the population of China.
38. Write an equation to represent the population of India.
39. Use elimination to find the year when the populations of China and India are predicted to be the same. What is the predicted population at that time?



40. **CRITICAL THINKING** The graphs of $Ax + By = 15$ and $Ax - By = 9$ intersect at $(2, 1)$. Find A and B .
41. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you use a system of equations to solve problems about weather?

Include the following in your answer:

- an explanation of how to use elimination to solve a system of equations, and
- a step-by-step solution of the Seward daylight problem.

Standardized Test Practice

42. If $2x - 3y = -9$ and $3x - 3y = -12$, what is the value of y ?
 (A) -3 (B) 1 (C) $(-3, 1)$ (D) $(1, -3)$
43. What is the solution of $4x + 2y = 8$ and $2x + 2y = 2$?
 (A) $(-2, 3)$ (B) $(3, 2)$ (C) $(3, -2)$ (D) $(12, -3)$

Maintain Your Skills

Mixed Review

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions. (Lesson 7-2)

44. $y = 5x$
 $x + 2y = 22$

45. $x = 2y + 3$
 $3x + 4y = -1$

46. $2y - x = -5$
 $4y - 3x = -1$

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. (Lesson 7-1)

47. $x - y = 3$
 $3x + y = 1$

48. $2x - 3y = 7$
 $3y = 7 + 2x$

49. $4x + y = 12$
 $x - 3 - \frac{1}{4}y$

50. Write an equation of a line that is parallel to the graph of $y = \frac{5}{4}x - 3$ and passes through the origin. (Lesson 5-6)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Use the Distributive Property to rewrite each expression without parentheses. (To review the **Distributive Property**, see Lesson 1-5.)

51. $2(3x + 4y)$ 52. $6(2a - 5b)$ 53. $-3(-2m + 3n)$ 54. $-5(4t - 2s)$

7-4

Elimination Using Multiplication

What You'll Learn

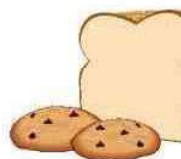
- Solve systems of equations by using elimination with multiplication.
- Determine the best method for solving systems of equations.

How can a manager use a system of equations to plan employee time?

The Finneytown Bakery is making peanut butter cookies and loaves of quick bread. The preparation and baking times for each are given in the table below.

For these two items, the management has allotted 800 minutes of employee time and 900 minutes of oven time. If c represents the number of batches of cookies and b represents the number of loaves of bread, the following system of equations can be used to determine how many of each to bake.

$$\begin{aligned} 20c + 10b &= 800 \\ 10c + 30b &= 900 \end{aligned}$$



	Cookies (per batch)	Bread (per loaf)
Preparation	20 min	10 min
Baking	10 min	30 min

ELIMINATION USING MULTIPLICATION Neither variable in the system above can be eliminated by simply adding or subtracting the equations. However, you can use the Multiplication Property of Equality so that adding or subtracting eliminates one of the variables.

Example 1 Multiply One Equation to Eliminate

Use elimination to solve the system of equations.

$$\begin{aligned} 3x + 4y &= 6 \\ 5x + 2y &= -4 \end{aligned}$$

Multiply the second equation by -2 so the coefficients of the y terms are additive inverses. Then add the equations.

$$\begin{array}{rcl} 3x + 4y & = & 6 \\ 5x + 2y & = & -4 \quad \text{Multiply by } -2. \\ \hline 3x + 4y & = & 6 \\ (+) -10x - 4y & = & 8 \\ \hline -7x & = & 14 \quad \text{Add the equations.} \\ \frac{-7x}{-7} & = & \frac{14}{-7} \quad \text{Divide each side by } -7. \\ x & = & -2 \quad \text{Simplify.} \end{array}$$

Now substitute -2 for x in either equation to find the value of y .

$$\begin{array}{rcl} 3x + 4y & = & 6 \quad \text{First equation} \\ 3(-2) + 4y & = & 6 \quad x = -2 \\ -6 + 4y & = & 6 \quad \text{Simplify.} \\ -6 + 4y + 6 & = & 6 + 6 \quad \text{Add 6 to each side.} \\ 4y & = & 12 \quad \text{Simplify.} \\ \frac{4y}{4} & = & \frac{12}{4} \quad \text{Divide each side by 4.} \\ y & = & 3 \quad \text{The solution is } (-2, 3). \end{array}$$

For some systems of equations, it is necessary to multiply each equation by a different number in order to solve the system by elimination. You can choose to eliminate either variable.

Example 2 Multiply Both Equations to Eliminate

Use elimination to solve the system of equations.

$$3x + 4y = -25$$

$$2x - 3y = 6$$

Method 1 Eliminate x .

$$\begin{array}{rcl} 3x + 4y = -25 & \text{Multiply by 2.} & 6x + 8y = -50 \\ 2x - 3y = 6 & \text{Multiply by } -3. & (-) -6x + 9y = -18 \\ \hline & & 17y = -68 \quad \text{Add the equations.} \\ & & \frac{17y}{17} = \frac{-68}{17} \quad \text{Divide each side by 17.} \\ & & y = -4 \quad \text{Simplify.} \end{array}$$

Now substitute -4 for y in either equation to find the value of x .

$$\begin{array}{rcl} 2x - 3y = 6 & \text{Second equation} & \\ 2x - 3(-4) = 6 & y = -4 & \\ 2x + 12 = 6 & \text{Simplify.} & \\ 2x + 12 - 12 = 6 - 12 & \text{Subtract 12 from each side.} & \\ 2x = -6 & \text{Simplify.} & \\ \frac{2x}{2} = \frac{-6}{2} & \text{Divide each side by 2.} & \\ x = -3 & \text{Simplify.} & \end{array}$$

The solution is $(-3, -4)$.

Method 2 Eliminate y .

$$\begin{array}{rcl} 3x + 4y = -25 & \text{Multiply by 3.} & 9x + 12y = -75 \\ 2x - 3y = 6 & \text{Multiply by 4.} & (+) 8x - 12y = 24 \\ \hline & & 17x = -51 \quad \text{Add the equations.} \\ & & \frac{17x}{17} = \frac{-51}{17} \quad \text{Divide each side by 17.} \\ & & x = -3 \quad \text{Simplify.} \end{array}$$

Now substitute -3 for x in either equation to find the value of y .

$$\begin{array}{rcl} 2x - 3y = 6 & \text{Second equation} & \\ 2(-3) - 3y = 6 & x = -3 & \\ -6 - 3y = 6 & \text{Simplify.} & \\ -6 - 3y + 6 = 6 + 6 & \text{Add 6 to each side.} & \\ -3y = 12 & \text{Simplify.} & \\ \frac{-3y}{-3} = \frac{12}{-3} & \text{Divide each side by } -3. & \\ y = -4 & \text{Simplify.} & \end{array}$$

The solution is $(-3, -4)$, which matches the result obtained with Method 1.

Study Tip

Using Multiplication

There are many other combinations of multipliers that could be used to solve the system in Example 2. For instance, the first equation could be multiplied by -2 and the second by 3 .

DETERMINE THE BEST METHOD You have learned five methods for solving systems of linear equations.

Concept Summary		Solving Systems of Equations	
Method	The Best Time to Use		
Graphing	to estimate the solution, since graphing usually does not give an exact solution		
Substitution	if one of the variables in either equation has a coefficient of 1 or -1		
Elimination Using Addition	if one of the variables has opposite coefficients in the two equations		
Elimination Using Subtraction	if one of the variables has the same coefficient in the two equations		
Elimination Using Multiplication	if none of the coefficients are 1 or -1 and neither of the variables can be eliminated by simply adding or subtracting the equations		

Example 3 Determine the Best Method

Determine the best method to solve the system of equations. Then solve the system.

$$\begin{aligned} 4x - 3y &= 12 \\ x + 2y &= 14 \end{aligned}$$

- For an exact solution, an algebraic method is best.
- Since neither the coefficients of x nor the coefficients of y are the same or additive inverses, you cannot use elimination using addition or subtraction.
- Since the coefficient of x in the second equation is 1, you can use the substitution method. You could also use elimination using multiplication.

The following solution uses substitution. Which method would you prefer?

$$x + 2y = 14 \quad \text{Second equation}$$

$$x + 2y - 2y = 14 - 2y \quad \text{Subtract } 2y \text{ from each side.}$$

$$x = 14 - 2y \quad \text{Simplify.}$$

$$4x - 3y = 12 \quad \text{First equation}$$

$$4(14 - 2y) - 3y = 12 \quad x = 14 - 2y$$

$$56 - 8y - 3y = 12 \quad \text{Distributive Property}$$

$$56 - 11y = 12 \quad \text{Combine like terms.}$$

$$56 - 11y - 56 = 12 - 56 \quad \text{Subtract 56 from each side.}$$

$$-11y = -44 \quad \text{Simplify.}$$

$$\frac{-11y}{-11} = \frac{-44}{-11} \quad \text{Divide each side by } -11.$$

$$y = 4 \quad \text{Simplify.}$$

$$x + 2y = 14 \quad \text{Second equation}$$

$$x + 2(4) = 14 \quad y = 4$$

$$x + 8 = 14 \quad \text{Simplify.}$$

$$x + 8 - 8 = 14 - 8 \quad \text{Subtract 8 from each side.}$$

$$x = 6 \quad \text{Simplify.}$$

The solution is $(6, 4)$.

Study Tip

Alternative Method

This system could also be solved easily by multiplying the second equation by 4 and then subtracting the equations.

More About



Transportation

About 203 million tons of freight are transported on the Ohio River each year making it the second most used commercial river in the United States.

Source: World Book Encyclopedia

Example 4 Write and Solve a System of Equations

TRANSPORTATION A coal barge on the Ohio River travels 24 miles upstream in 3 hours. The return trip takes the barge only 2 hours. Find the rate of the barge in still water.

Let b = the rate of the barge in still water and c = the rate of the current. Use the formula $\text{rate} \times \text{time} = \text{distance}$, or $rt = d$.

	r	t	d	$rt = d$
Downstream	$b + c$	2	24	$2b + 2c = 24$
Upstream	$b - c$	3	24	$3b - 3c = 24$

This system cannot easily be solved using substitution. It cannot be solved by just adding or subtracting the equations.

The best way to solve this system is to use elimination using multiplication. Since the problem asks for b , eliminate c .

$$\begin{array}{rcl}
 2b + 2c = 24 & \text{Multiply by 3.} & 6b + 6c = 72 \\
 3b - 3c = 24 & \text{Multiply by 2.} & (+) 6b - 6c = 48 \\
 \hline
 12b & = & 120 \quad \text{Add the equations.} \\
 \frac{12b}{12} = \frac{120}{12} & & \text{Divide each side by 12.} \\
 b = 10 & & \text{Simplify.}
 \end{array}$$

The rate of the barge in still water is 10 miles per hour.

Check for Understanding

Concept Check

1. Explain why multiplication is sometimes needed to solve a system of equations by elimination.
2. **OPEN ENDED** Write a system of equations that could be solved by multiplying one equation by 5 and then adding the two equations together to eliminate one variable.
3. **Describe** two methods that could be used to solve the following system of equations. Which method do you prefer? Explain.

$$\begin{array}{l}
 a - b = 5 \\
 2a + 3b = 15
 \end{array}$$

Guided Practice

Use elimination to solve each system of equations.

$$\begin{array}{l}
 4. \quad 2x - y = 6 \\
 \quad \quad 3x + 4y = -2
 \end{array}$$

$$\begin{array}{l}
 5. \quad x + 5y = 4 \\
 \quad \quad 3x - 7y = -10
 \end{array}$$

$$\begin{array}{l}
 6. \quad 4x + 7y = 6 \\
 \quad \quad 6x + 5y = 20
 \end{array}$$

$$\begin{array}{l}
 7. \quad 4x + 2y = 10.5 \\
 \quad \quad 2x + 3y = 10.75
 \end{array}$$

Determine the best method to solve each system of equations. Then solve the system.

$$\begin{array}{l}
 8. \quad 4x + 3y = 19 \\
 \quad \quad 3x - 4y = 8
 \end{array}$$

$$\begin{array}{l}
 9. \quad 3x - 7y = 6 \\
 \quad \quad 2x + 7y = 4
 \end{array}$$

$$\begin{array}{l}
 10. \quad y = 4x + 11 \\
 \quad \quad 3x - 2y = -7
 \end{array}$$

$$\begin{array}{l}
 11. \quad 5x - 2y = 12 \\
 \quad \quad 3x - 2y = -2
 \end{array}$$

- Application** 12. **BUSINESS** The owners of the River View Restaurant have hired enough servers to handle 17 tables of customers, and the fire marshal has approved the restaurant for a limit of 56 customers. How many two-seat tables and how many four-seat tables should the owners purchase?

Practice and Apply

Homework Help

For Exercises	See Examples
13–26	1, 2
27–38	3
39–43	4

Extra Practice

See page 836.

Use elimination to solve each system of equations.

13. $-5x + 3y = 6$
 $x - y = 4$
14. $x + y = 3$
 $2x - 3y = 16$
15. $2x + y = 5$
 $3x - 2y = 4$
16. $4x - 3y = 12$
 $x + 2y = 14$
17. $5x - 2y = -15$
 $3x + 8y = 37$
18. $8x - 3y = -11$
 $2x - 5y = 27$
19. $4x - 7y = 10$
 $3x + 2y = -7$
20. $2x - 3y = 2$
 $5x + 4y = 28$
21. $1.8x - 0.3y = 14.4$
 $x - 0.6y = 2.8$
22. $0.4x + 0.5y = 2.5$
 $1.2x - 3.5y = 2.5$
23. $3x - \frac{1}{2}y = 10$
 $5x + \frac{1}{4}y = 8$
24. $2x + \frac{2}{3}y = 4$
 $x - \frac{1}{2}y = 7$

25. Seven times a number plus three times another number equals negative one. The sum of the two numbers is negative three. What are the numbers?
26. Five times a number minus twice another number equals twenty-two. The sum of the numbers is three. Find the numbers.

Determine the best method to solve each system of equations. Then solve the system.

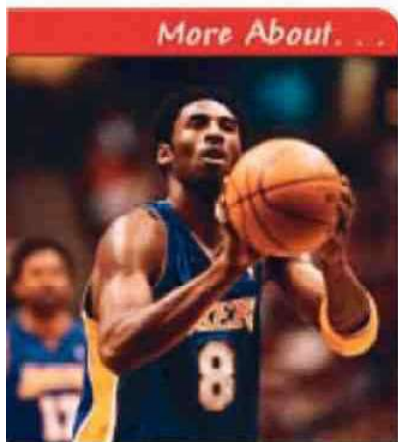
27. $3x - 4y = -10$
 $5x + 8y = -2$
28. $9x - 8y = 42$
 $4x + 8y = -16$
29. $y = 3x$
 $3x + 4y = 30$
30. $x = 4y + 8$
 $2x - 8y = -3$
31. $2x - 3y = 12$
 $x + 3y = 12$
32. $4x - 2y = 14$
 $y = x$
33. $x - y = 2$
 $5x + 3y = 18$
34. $y = 2x + 9$
 $2x - y = -9$
35. $6x - y = 9$
 $6x - y = 11$
36. $x = 8y$
 $2x + 3y = 38$
37. $\frac{2}{3}x - \frac{1}{2}y = 14$
 $\frac{5}{6}x - \frac{1}{2}y = 18$
38. $\frac{1}{2}x - \frac{2}{3}y = \frac{7}{3}$
 $\frac{3}{2}x + 2y = -25$

39. **BASKETBALL** In basketball, a free throw is 1 point and a field goal is either 2 points or 3 points. In the 2000–2001 season, Kobe Bryant scored a total of 1938 points. The total number of 2-point field goals and 3-point field goals was 701. Use the information at the left to find the number of Kobe Bryant's 2-point field goals and 3-point field goals that season.



Online Research Data Update What are the current statistics for Kobe Bryant and other players? Visit www.algebra1.com/data_update to learn more.

40. **CRITICAL THINKING** The solution of the system $4x + 5y = 2$ and $6x - 2y = b$ is $(3, a)$. Find the values of a and b .
41. **CAREERS** Mrs. Henderson discovered that she had accidentally reversed the digits of a test and shorted a student 36 points. Mrs. Henderson told the student that the sum of the digits was 14 and agreed to give the student his correct score plus extra credit if he could determine his actual score without looking at his test. What was his actual score on the test?



Basketball

In the 2000–2001 season, Kobe Bryant ranked 18th in the NBA in free-throw percentage. He made 475 of the 557 free throws that he attempted.

Source: NBA



www.algebra1.com/self_check_quiz



42. **NUMBER THEORY** The sum of the digits of a two-digit number is 14. If the digits are reversed, the new number is 18 less than the original number. Find the original number.
43. **TRANSPORTATION** Traveling against the wind, a plane flies 2100 miles from Chicago to San Diego in 4 hours and 40 minutes. The return trip, traveling with a wind that is twice as fast, takes 4 hours. Find the rate of the plane in still air.
44. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a manager use a system of equations to plan employee time?

Include the following in your answer:

- a demonstration of how to solve the system of equations concerning the cookies and bread, and
- an explanation of how a restaurant manager would schedule oven and employee time.

Standardized Test Practice

45. If $5x + 3y = 12$ and $4x - 5y = 17$, what is the value of y ?
 (A) -1 (B) 3 (C) (-1, 3) (D) (3, -1)
46. Determine the number of solutions of the system $x + 2y = -1$ and $2x + 4y = -2$.
 (A) 0 (B) 1 (C) 2 (D) infinitely many

Maintain Your Skills

Mixed Review

Use elimination to solve each system of equations. (Lesson 7-3)

47. $x + y = 8$
 $x - y = 4$

48. $2r + s = 5$
 $r - s = 1$

49. $x + y = 18$
 $x + 2y = 25$

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions. (Lesson 7-2)

50. $2x + 3y = 3$
 $x = -3y$

51. $x + y = 0$
 $3x + y = -8$

52. $x - 2y = 7$
 $-3x + 6y = -21$

53. **CAREERS** A store manager is paid \$32,000 a year plus 4% of the revenue the store makes above quota. What is the amount of revenue above quota needed for the manager to have an annual income greater than \$45,000? (Lesson 6-3)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Graph each inequality.

(To review **graphing inequalities**, see Lesson 6-6.)

54. $y \geq x - 7$

55. $x + 3y \geq 9$

56. $-y \leq x$

57. $-3x + y \geq -1$

Practice Quiz 2

Lessons 7-3 and 7-4

Use elimination to solve each system of equations. (Lessons 7-3 and 7-4)

1. $5x + 4y = 2$
 $3x - 4y = 14$

2. $2x - 3y = 13$
 $2x + 2y = -2$

3. $6x - 2y = 24$
 $3x + 4y = 27$

4. $5x + 2y = 4$
 $10x + 4y = 9$

5. The price of a cellular telephone plan is based on peak and nonpeak service. Kelsey used 45 peak minutes and 50 nonpeak minutes and was charged \$27.75. That same month, Mitch used 70 peak minutes and 30 nonpeak minutes for a total charge of \$36. What are the rates per minute for peak and nonpeak time? (Lesson 7-4)

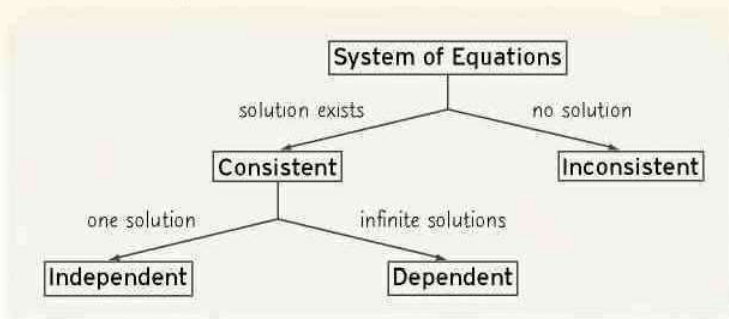


Reading Mathematics

Making Concept Maps

After completing a chapter, it is wise to review each lesson's main topics and vocabulary. In Lesson 7-1, the new vocabulary words were *system of equations*, *consistent*, *inconsistent*, *independent*, and *dependent*. They are all related in that they explain how many and what kind of solutions a system of equations has.

A graphic organizer called a *concept map* is a convenient way to show these relationships. A concept map is shown below for the vocabulary words for Lesson 7-1. The main ideas are placed in boxes. Any information that describes how to move from one box to the next is placed along the arrows.

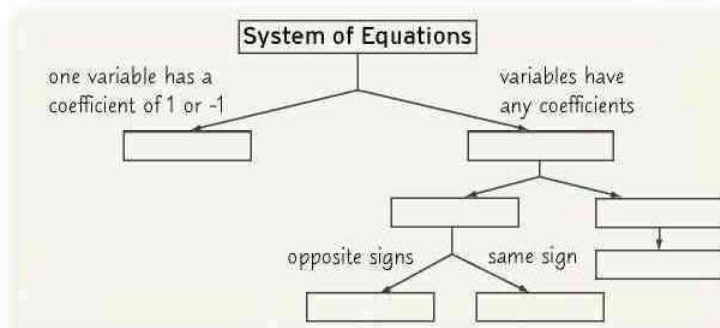


Concept maps are used to organize information. They clearly show how ideas are related to one another. They also show the flow of mental processes needed to solve problems.

Reading to Learn

Review Lessons 7-2, 7-3, and 7-4.

1. Write a couple of sentences describing the information in the concept map above.
2. How do you decide whether to use substitution or elimination? Give an example of a system that you would solve using each method.
3. How do you decide whether to multiply an equation by a factor?
4. How do you decide whether to add or subtract two equations?
5. Copy and complete the concept map below for solving systems of equations by using either substitution or elimination.



7-5

Graphing Systems of Inequalities

What You'll Learn

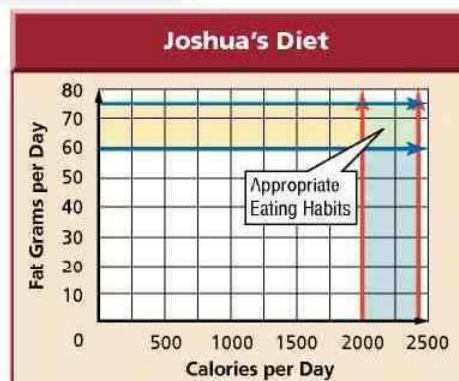
- Solve systems of inequalities by graphing.
- Solve real-world problems involving systems of inequalities.

Vocabulary

- system of inequalities

How can you use a system of inequalities to plan a sensible diet?

Joshua watches what he eats. His doctor told him to eat between 2000 and 2400 Calories per day. The doctor also wants him to keep his daily fat intake between 60 and 75 grams. The graph indicates the appropriate amounts of Calories and fat for Joshua. The graph is of a system of inequalities. He should try to keep his Calorie and fat intake to amounts represented in the green section.



SYSTEMS OF INEQUALITIES To solve a **system of inequalities**, you need to find the ordered pairs that satisfy all the inequalities involved. One way to do this is to graph the inequalities on the same coordinate plane. The solution set is represented by the intersection, or overlap, of the graphs.

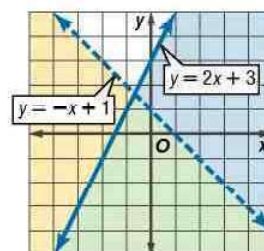
Example 1 Solve by Graphing

Solve the system of inequalities by graphing.

$$y < -x + 1$$

$$y \leq 2x + 3$$

The solution includes the ordered pairs in the intersection of the graphs of $y < -x + 1$ and $y \leq 2x + 3$. This region is shaded in green at the right. The graphs of $y = -x + 1$ and $y = 2x + 3$ are boundaries of this region. The graph of $y = -x + 1$ is dashed and is *not* included in the graph of $y < -x + 1$. The graph of $y = 2x + 3$ is included in the graph of $y \leq 2x + 3$.



Study Tip

Look Back

To review **graphing linear inequalities**, see Lesson 6-6.

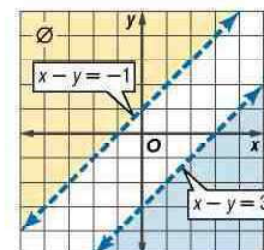
Example 2 No Solution

Solve the system of inequalities by graphing.

$$x - y < -1$$

$$x - y > 3$$

The graphs of $x - y = -1$ and $x - y = 3$ are parallel lines. Because the two regions have no points in common, the system of inequalities has no solution.



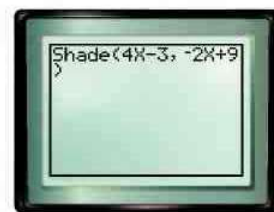
You can use a TI-83 Plus to solve systems of inequalities.



Graphing Calculator Investigation

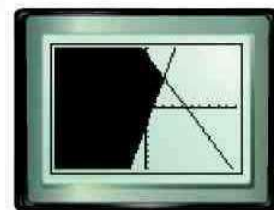
Graphing Systems of Inequalities

To graph the system $y \geq 4x - 3$ and $y \leq -2x + 9$ on a TI-83 Plus, select the SHADE feature in the DRAW menu. Enter the function that is the lower boundary of the region to be shaded, followed by the upper boundary. (Note that inequalities that have $>$ or $\geq$ are lower boundaries and inequalities that have $<$ or $\leq$ are upper boundaries.)



Think and Discuss

1. To graph the system $y \leq 3x + 1$ and $y \geq -2x - 5$ on a graphing calculator, which function should you enter first?
2. Use a graphing calculator to graph the system $y \leq 3x + 1$ and $y \geq -2x - 5$.
3. Explain how you could use a graphing calculator to graph the system $2x + y \geq 7$ and $x - 2y \geq 5$.
4. Use a graphing calculator to graph the system $2x + y \geq 7$ and $x - 2y \geq 5$.



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

REAL-WORLD PROBLEMS In real-life problems involving systems of inequalities, sometimes only whole-number solutions make sense.

Example 3 Use a System of Inequalities to Solve a Problem

COLLEGE The middle 50% of first-year students attending Florida State University score between 520 and 620, inclusive, on the verbal portion of the SAT and between 530 and 630, inclusive, on the math portion. Graph the scores that a student would need to be in the middle 50% of FSU freshmen.

Words

The verbal score is between 520 and 620, inclusive. The math score is between 530 and 630, inclusive.

Variables

If v = the verbal score and m = the math score, the following inequalities represent the middle 50% of Florida State University freshmen.

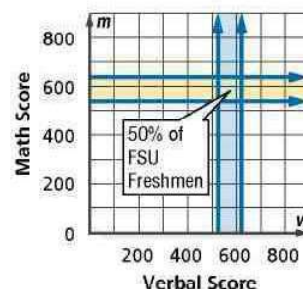
Inequalities

The verbal score is between 520 and 620, inclusive.

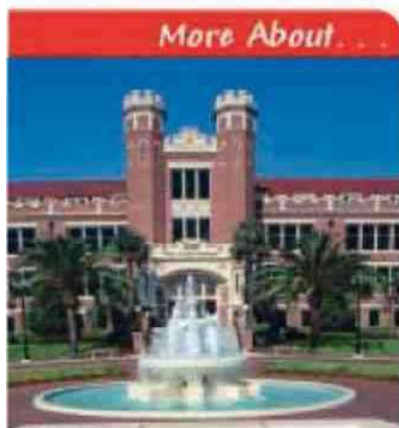
$$520 \leq v \leq 620$$

The math score is between 530 and 630, inclusive.

$$530 \leq m \leq 630$$



The solution is the set of all ordered pairs whose graphs are in the intersection of the graphs of these inequalities. However, since SAT scores are whole numbers, only whole-number solutions make sense in this problem.



College

FSU is the most wired campus in Florida and has been recently ranked the 10th most technologically connected university in the nation.

Source: www.fsu.edu

Example 4 Use a System of Inequalities

AGRICULTURE To ensure a growing season of sufficient length, Mr. Hobson has at most 16 days left to plant his corn and soybean crops. He can plant corn at a rate of 250 acres per day and soybeans at a rate of 200 acres per day. If he has at most 3500 acres available, how many acres of each type of crop can he plant?

Let c = the number of days that corn will be planted and s = the number of days that soybeans will be planted. Since both c and s represent a number of days, neither can be a negative number. The following system of inequalities can be used to represent the conditions of this problem.

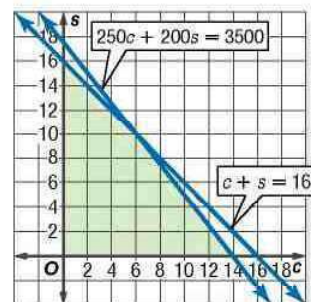
$$c \geq 0$$

$$s \geq 0$$

$$c + s \leq 16$$

$$250c + 200s \leq 3500$$

The solution is the set of all ordered pairs whose graphs are in the intersection of the graphs of these inequalities. This region is shown in green at the right. Only the portion of the region in the first quadrant is used since $c \geq 0$ and $s \geq 0$.

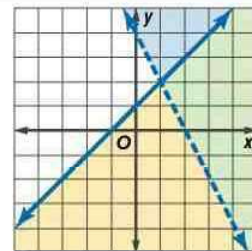


Any point in this region is a possible solution. For example, since $(7, 8)$ is a point in the region, Mr. Hobson could plant corn for 7 days and soybeans for 8 days. In this case, he would use 15 days to plant $250(7)$ or 1750 acres of corn and $200(8)$ or 1600 acres of soybeans.

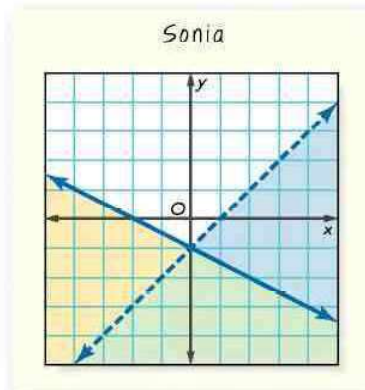
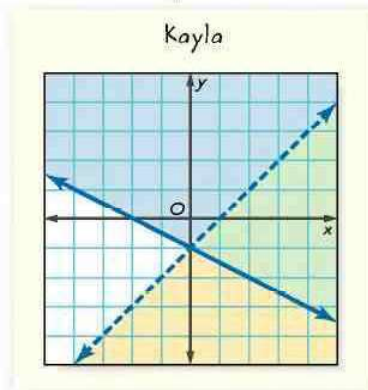
Check for Understanding

- Concept Check**
- OPEN ENDED** Draw the graph of a system of inequalities that has no solution.
 - Determine** which of the following ordered pairs represent a solution of the system of inequalities graphed at the right.

a. $(3, 1)$	b. $(-1, -3)$
c. $(2, 3)$	d. $(4, -2)$
e. $(3, -2)$	f. $(1, 4)$



- FIND THE ERROR** Kayla and Sonia are solving the system of inequalities $x + 2y \geq -2$ and $x - y > 1$.



Who is correct? Explain your reasoning.

Guided Practice Solve each system of inequalities by graphing.

4. $x > 5$
 $y \leq 4$
5. $y > 3$
 $y > -x + 4$
6. $y \leq -x + 3$
 $y \leq x + 3$
7. $2x + y \geq 4$
 $y \leq -2x - 1$
8. $2y + x < 6$
 $3x - y > 4$
9. $x - 2y \leq 2$
 $3x + 4y \leq 12$
 $x \geq 0$

Application HEALTH For Exercises 10 and 11, use the following information.

Natasha walks and jogs at least 3 miles every day. Natasha walks 4 miles per hour and jogs 8 miles per hour. She only has a half-hour to exercise.

10. Draw a graph of the possible amounts of time she can spend walking and jogging.
11. List three possible solutions.

Practice and Apply

Homework Help

For Exercises	See Examples
12–28	1–2
29–31, 33–35	3–4

Extra Practice

See page 836.

Career Choices



Visual Artist

Visual artists create art to communicate ideas. The work of fine artists is made for display. Illustrators and graphic designers produce art for clients, such as advertising and publishing companies.

Online Research

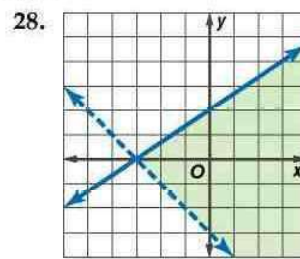
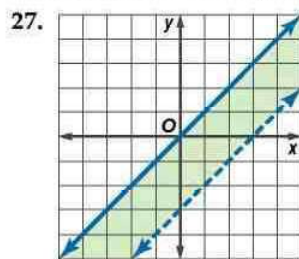
For information about a career as a visual artist, visit:

www.algebra1.com/careers

Solve each system of inequalities by graphing.

12. $y < 0$
 $x \geq 0$
13. $x > -4$
 $y \leq -1$
14. $y \geq -2$
 $y - x < 1$
15. $x \geq 2$
 $y + x \leq 5$
16. $x \leq 3$
 $x + y > 2$
17. $y \geq 2x + 1$
 $y \leq -x + 1$
18. $y < 2x + 1$
 $y \geq -x + 3$
19. $y - x < 1$
 $y - x > 3$
20. $y - x < 3$
 $y - x \geq 2$
21. $2x + y \leq 4$
 $3x - y \geq 6$
22. $3x - 4y < 1$
 $x + 2y \leq 7$
23. $x + y > 4$
 $-2x + 3y < -12$
24. $2x + y \geq -4$
 $-5x + 2y < 1$
25. $y \leq x + 3$
 $2x - 7y \leq 4$
 $3x + 2y \leq 6$
26. $x < 2$
 $4y > x$
 $2x - y > -9$
 $x + 3y < 9$

Write a system of inequalities for each graph.



ART For Exercises 29 and 30, use the following information.

A painter has exactly 32 units of yellow dye and 54 units of blue dye. She plans to mix the dyes to make two shades of green. Each gallon of the lighter shade of green requires 4 units of yellow dye and 1 unit of blue dye. Each gallon of the darker shade of green requires 1 unit of yellow dye and 6 units of blue dye.

29. Make a graph showing the numbers of gallons of the two greens she can make.
30. List three possible solutions.

31. **HEALTH** The LDL or “bad” cholesterol of a teenager should be less than 110. The HDL or “good” cholesterol of a teenager should be between 35 and 59. Make a graph showing appropriate levels of cholesterol for a teenager.

32. **CRITICAL THINKING** Write a system of inequalities that is equivalent to $|x| \leq 4$.

MANUFACTURING For Exercises 33 and 34, use the following information. The Natural Wood Company has machines that sand and varnish desks and tables. The table gives the time requirements of the machines.

Machine	Hours per Desk	Hours per Table	Total Hours Available Each Week
Sanding	2	1.5	31
Varnishing	1.5	1	22

33. Make a graph showing the number of desks and the number of tables that can be made in a week.
34. List three possible solutions.
35. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you use a system of inequalities to plan a sensible diet?

Include the following in your answer:

- two appropriate Calorie and fat intakes for a day, and
- the system of inequalities that is represented by the graph.



Graphing Calculator

GRAPHING SYSTEMS OF INEQUALITIES Use a graphing calculator to solve each system of inequalities. Sketch the results.

36. $y \leq x + 9$
 $y \geq -x - 4$

37. $y \leq 2x + 10$
 $y \geq 7x + 15$

38. $3x - y \leq 6$
 $x - y \geq -1$

39. Which ordered pair does *not* satisfy the system $x + 2y > 5$ and $3x - y < -2$?

(A) $(-3, 7)$

(B) $(0, 5)$

(C) $(-1, 4)$

(D) $(0, 2.5)$

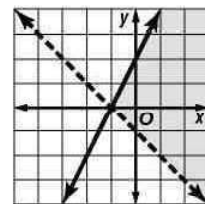
40. Which system of inequalities is represented by the graph?

(A) $y \leq 2x + 2$
 $y > -x - 1$

(B) $y \geq 2x + 2$
 $y < -x - 1$

(C) $y < 2x + 2$
 $y \leq -x - 1$

(D) $y > 2x + 2$
 $y \leq -x - 1$



Maintain Your Skills

Mixed Review

Use elimination to solve each system of equations. (Lessons 7-3 and 7-4)

41. $2x + 3y = 1$
 $4x - 5y = 13$

42. $5x - 2y = -3$
 $3x + 6y = -9$

43. $-3x + 2y = 12$
 $2x - 3y = -13$

44. $6x - 2y = 4$
 $5x - 3y = -2$

45. $2x + 5y = 13$
 $3x - 5y = -18$

46. $3x - y = 6$
 $3x + 2y = 15$

Write an equation of the line that passes through each point with the given slope. (Lesson 5-4)

47. $(4, -1)$, $m = 2$

48. $(1, 0)$, $m = -6$

49. $(5, -2)$, $m = \frac{1}{3}$

WebQuest Internet Project

The Spirit of the Games

It's time to complete your project. Use the information and data you have gathered about the Olympics to prepare a portfolio or Web page. Be sure to include graphs and/or tables in your project.

www.algebra1.com/webquest

Chapter 7

Study Guide and Review

Vocabulary and Concept Check

consistent (p. 369)
dependent (p. 369)

elimination (p. 382)
inconsistent (p. 369)

independent (p. 369)
substitution (p. 376)

system of equations (p. 369)
system of inequalities (p. 394)

Choose the correct term to complete each statement.

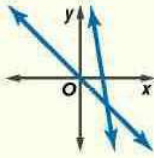
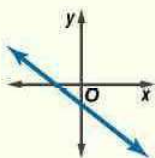
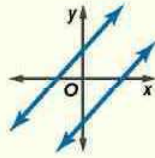
1. If a system of equations has exactly one solution, it is (*dependent*, *independent*).
2. If the graph of a system of equations is parallel lines, the system is (*consistent*, *inconsistent*).
3. A system of equations that has infinitely many solutions is (*dependent*, *independent*).
4. If the equations in a system have the same slope and different intercepts, the graph of the system is (*intersecting lines*, *parallel lines*).
5. If a system of equations has the same slope and intercepts, the system has (*exactly one*, *infinitely many*) solution(s).
6. The solution of a system of equations is $(3, -5)$. The system is (*consistent*, *inconsistent*).

Lesson-by-Lesson Review

7-1 Graphing Systems of Inequalities

See pages
369–374.

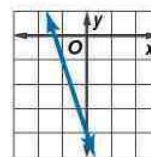
Concept Summary

	Intersecting Lines	Same Line	Parallel Lines
Graph of a System			
Number of Solutions	exactly one solution	infinitely many	no solutions
Terminology	consistent and independent	consistent and dependent	inconsistent

Example

Graph the system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

$$\begin{aligned} 3x + y &= -4 \\ 6x + 2y &= -8 \end{aligned}$$



When the lines are graphed, they coincide. There are infinitely many solutions.

Exercises Graph each system of equations. Then determine whether the system of equations has *one* solution, *no* solution, or *infinitely many* solutions. If the system has one solution, name it. See Example 2 on page 370.

7. $\begin{aligned} x - y &= 9 \\ x + y &= 11 \end{aligned}$

8. $\begin{aligned} 9x + 2 &= 3y \\ y - 3x &= 8 \end{aligned}$

9. $\begin{aligned} 2x - 3y &= 4 \\ 6y &= 4x - 8 \end{aligned}$

10. $\begin{aligned} 3x - y &= 8 \\ 3x &= 4 - y \end{aligned}$

7-2 Substitution

See pages
376–381.

Concept Summary

- In a system of equations, solve one equation for a variable, and then substitute that expression into the second equation to solve.

Example

Use substitution to solve the system of equations.

$$\begin{aligned} y &= x - 1 \\ 4x - y &= 19 \end{aligned}$$

Since $y = x - 1$, substitute $x - 1$ for y in the second equation.

$$4x - y = 19 \quad \text{Second equation}$$

$$4x - (x - 1) = 19 \quad y = x - 1$$

$$4x - x + 1 = 19 \quad \text{Distributive Property}$$

$$3x + 1 = 19 \quad \text{Combine like terms.}$$

$$3x = 18 \quad \text{Subtract 1 from each side.}$$

$$x = 6 \quad \text{Divide each side by 3.}$$

Use $y = x - 1$ to find the value of y .

$$y = x - 1 \quad \text{First equation}$$

$$y = 6 - 1 \quad x = 6$$

$$y = 5 \quad \text{The solution is } (6, 5).$$

Exercises Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solutions or *infinitely many* solutions. See Examples 1–3 on pages 377 and 378.

$$\begin{aligned} 11. \quad 2m + n &= 1 \\ m - n &= 8 \end{aligned}$$

$$\begin{aligned} 12. \quad x &= 3 - 2y \\ 2x + 4y &= 6 \end{aligned}$$

$$\begin{aligned} 13. \quad 3x - y &= 1 \\ 2x + 4y &= 3 \end{aligned}$$

$$\begin{aligned} 14. \quad 0.6m - 0.2n &= 0.9 \\ n &= 4.5 - 3m \end{aligned}$$

7-3 Elimination Using Addition and Subtraction

See pages
382–386.

Concept Summary

- Sometimes adding or subtracting two equations will eliminate one variable.

Example

Use elimination to solve the system of equations.

$$\begin{aligned} 2m - n &= 4 \\ m + n &= 2 \end{aligned}$$

You can eliminate the n terms by adding the equations.

$$2m - n = 4 \quad \text{Write the equations in column form and add.}$$

$$\begin{array}{r} (+) \, 2m - n = 4 \\ \quad m + n = 2 \\ \hline \end{array}$$

$$3m = 6 \quad \text{Notice the variable } n \text{ is eliminated.}$$

$$m = 2 \quad \text{Divide each side by 3.}$$

Now substitute 2 for m in either equation to find n .

$$m + n = 2 \quad \text{Second equation}$$

$$2 + n = 2 \quad m = 2$$

$$2 + n - 2 = 2 - 2 \quad \text{Subtract 2 from each side.}$$

$$n = 0 \quad \text{Simplify.}$$

The solution is $(2, 0)$.

Exercises Use elimination to solve each system of equations.

See Examples 1–3 on pages 382 and 383.

$$\begin{array}{llll} 15. \begin{cases} x + 2y = 6 \\ x - 3y = -4 \end{cases} & 16. \begin{cases} 2m - n = 5 \\ 2m + n = 3 \end{cases} & 17. \begin{cases} 3x - y = 11 \\ x + y = 5 \end{cases} & 18. \begin{cases} 3x + 1 = -7y \\ 6x + 7y = 0 \end{cases} \end{array}$$

7-4 Elimination Using Multiplication

See pages
387–392.

Concept Summary

- Multiplying one equation by a number or multiplying each equation by a different number is a strategy that can be used to solve a system of equations by elimination.
- There are five methods for solving systems of equations.

Method	The Best Time to Use
Graphing	to estimate the solution, since graphing usually does not give an exact solution
Substitution	if one of the variables in either equation has a coefficient of 1 or -1
Elimination Using Addition	if one of the variables has opposite coefficients in the two equations
Elimination Using Subtraction	if one of the variables has the same coefficient in the two equations
Elimination Using Multiplication	if none of the coefficients are 1 or -1 and neither of the variables can be eliminated by simply adding or subtracting the equations

Example Use elimination to solve the system of equations.

$$x + 2y = 8$$

$$3x + y = 1.5$$

Multiply the second equation by -2 so the coefficients of the y terms are additive inverses. Then add the equations.

$$x + 2y = 8$$

$$3x + y = 1.5$$

Multiply by -2 .

$$x + 2y = 8$$

$$(-2) \cdot (3x + y) = (-2) \cdot 1.5$$

$$-5x = 5$$

Add the equations.

$$\frac{-5x}{-5} = \frac{5}{-5}$$

Divide each side by -5 .

$$x = -1$$

Simplify.

(continued on the next page)



- Extra Practice, see pages 835–836.
- Mixed Problem Solving, see page 859.

$$x + 2y = 8$$

First equation

$$-1 + 2y = 8$$

 $x = -1$

$$-1 + 2y + 1 = 8 + 1 \quad \text{Add 1 to each side.}$$

$$2y = 9$$

Simplify.

$$\frac{2y}{2} = \frac{9}{2}$$

Divide each side by 2.

$$y = 4.5$$

Simplify.

The solution is $(-1, 4.5)$.**Exercises** Use elimination to solve each system of equations.

See Examples 1 and 2 on pages 387 and 388.

19. $x - 5y = 0$

$2x - 3y = 7$

21. $2x + 3y = 8$

$x - y = 2$

20. $x - 2y = 5$

$3x - 5y = 8$

22. $-5x + 8y = 21$

$10x + 3y = 15$

Determine the best method to solve each system of equations. Then solve the system. See Example 3 on page 389.

23. $y = 2x$

$x + 2y = 8$

25. $3x + 5y = 2x$

$x + 3y = y$

24. $9x + 8y = 7$

$18x - 15y = 14$

26. $2x + y = 3x - 15$

$x + 5 = 4y + 2x$

7-5See pages
394–398.**Graphing Systems of Inequalities****Concept Summary**

- Graph each inequality on a coordinate plane to determine the intersection of the graphs.

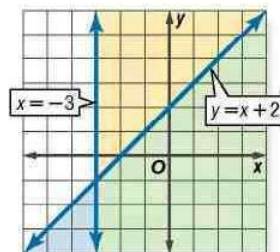
Example

Solve the system of inequalities.

$x \geq -3$

$y \leq x + 2$

The solution includes the ordered pairs in the intersection of the graphs $x \geq -3$ and $y \leq x + 2$. This region is shaded in green. The graphs of $x \geq -3$ and $y \leq x + 2$ are boundaries of this region.

**Exercises** Solve each system of inequalities by graphing.

See Examples 1 and 2 on page 394.

27. $y < 3x$

$x + 2y \geq -21$

29. $2x + y < 9$

$x + 11y < -6$

28. $y > -x - 1$

$y \leq 2x + 1$

30. $x > 1$

$y + x \leq 3$

Chapter 7

Practice Test

Vocabulary and Concepts

Choose the letter that best matches each description.

- a system of equations with two parallel lines
- a system of equations with at least one ordered pair that satisfies both equations
- a system of equations may be solved using this method

- consistent
 - elimination
 - inconsistent

Skills and Applications

Graph each system of equations. Then determine whether the system has *no solution*, *one solution*, or *infinitely many solutions*. If the system has one solution, name it.

4. $y = x + 2$
 $y = 2x + 7$

5. $x + 2y = 11$
 $x = 14 - 2y$

6. $3x + y = 5$
 $2y - 10 = -6x$

Use substitution or elimination to solve each system of equations.

7. $2x + 5y = 16$
 $5x - 2y = 11$

8. $y + 2x = -1$
 $y - 4 = -2x$

9. $2x + y = -4$
 $5x + 3y = -6$

10. $y = 7 - x$
 $x - y = -3$

11. $x = 2y - 7$
 $y - 3x = -9$

12. $x + y = 10$
 $x - y = 2$

13. $3x - y = 11$
 $x + 2y = -36$

14. $3x + y = 10$
 $3x - 2y = 16$

15. $5x - 3y = 12$
 $-2x + 3y = -3$

16. $2x + 5y = 12$
 $x - 6y = -11$

17. $x + y = 6$
 $3x - 3y = 13$

18. $3x + \frac{1}{3}y = 10$
 $2x - \frac{5}{3}y = 35$

19. **NUMBER THEORY** The units digit of a two-digit number exceeds twice the tens digit by 1. Find the number if the sum of its digits is 10.

20. **GEOMETRY** The difference between the length and width of a rectangle is 7 centimeters. Find the dimensions of the rectangle if its perimeter is 50 centimeters.

Solve each system of inequalities by graphing.

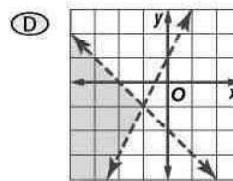
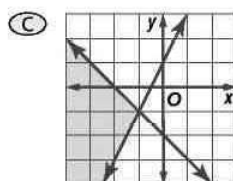
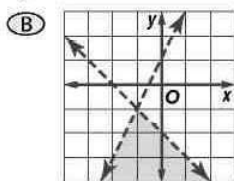
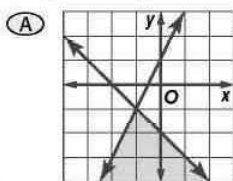
21. $y > -4$
 $y < -1$

22. $y \leq 3$
 $y > -x + 2$

23. $x \leq 2y$
 $2x + 3y \leq 7$

24. **FINANCE** Last year, Jodi invested \$10,000, part at 6% annual interest and the rest at 8% annual interest. If she received \$760 in interest at the end of the year, how much did she invest at each rate?

25. **STANDARDIZED TEST PRACTICE** Which graph represents the system of inequalities $y > 2x + 1$ and $y < -x - 2$?



Chapter 7

Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. What is the value of x in $4x - 2(x - 2) - 8 = 0$?
(Lesson 3-4)

(A) -2 (B) 2
(C) 5 (D) 6

2. Noah paid \$17.11 for a CD, including tax. If the tax rate is 7%, then what was the price of the CD before tax? (Lesson 3-5)

(A) \$10.06 (B) \$11.98
(C) \$15.99 (D) \$17.04

3. What is the range of $f(x) = 2x - 3$ when the domain is $\{3, 4, 5\}$? (Lesson 4-3)

(A) $\{0, 1, 2\}$ (B) $\{3, 5, 7\}$
(C) $\{6, 8, 10\}$ (D) $\{9, 11, 13\}$

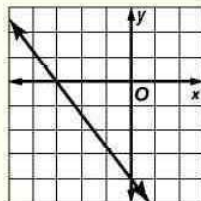
4. Jolene kept a log of the numbers of birds that visited a birdfeeder over periods of several hours. On the table below, she recorded the number of hours she watched and the cumulative number of birds that she saw each session. Which equation best represents this data set shown in the table? (Lesson 4-8)

Number of hours, x	1	3	4	6
Number of birds, y	6	14	18	26

(A) $y = x + 5$ (B) $y = 3x + 3$
(C) $y = 3x + 5$ (D) $y = 4x + 2$

5. Which equation describes the graph?
(Lesson 5-3)

(A) $3y - 4x = -12$
(B) $4y + 3x = -16$
(C) $3y + 4x = -12$
(D) $3y + 4x = -9$



6. Which equation represents a line parallel to the line given by $y - 3x = 6$? (Lesson 5-6)

(A) $y = -3x + 4$ (B) $y = 3x - 2$
(C) $y = \frac{1}{3}x + 6$ (D) $y = -\frac{1}{3}x + 4$

7. Tamika has \$185 in her bank account. She needs to deposit enough money so that she can withdraw \$230 for her car payment and still have at least \$200 left in the account. Which inequality describes d , the amount she needs to deposit? (Lesson 6-1)

(A) $d(185 - 230) \geq 200$
(B) $185 - 230d \geq 200$
(C) $185 + 230 + d \geq 200$
(D) $185 + d - 230 \geq 200$

8. The perimeter of a rectangular garden is 68 feet. The length of the garden is 4 more than twice the width. Which system of equations will determine the length ℓ and the width w of the garden? (Lesson 7-2)

(A) $2\ell + 2w = 68$ (B) $2\ell + 2w = 68$
 $\ell = 4 - 2w$ $\ell = 2w + 4$
(C) $2 + 2w = 68$ (D) $2\ell + 2w = 68$
 $2\ell - w = 4$ $w = 2\ell + 4$

9. Ernesto spent a total of \$64 for a pair of jeans and a shirt. The jeans cost \$6 more than the shirt. What was the cost of the jeans? (Lesson 7-2)

(A) \$26 (B) \$29
(C) \$35 (D) \$58

10. What is the value of y in the following system of equations? (Lesson 7-3)

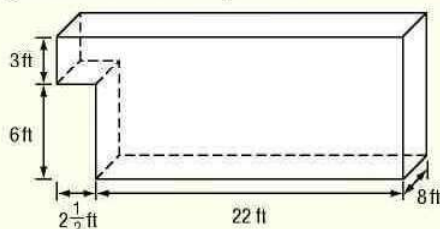
$$\begin{aligned} 3x + 4y &= 8 \\ 3x + 2y &= -2 \end{aligned}$$

(A) -2 (B) 4
(C) 5 (D) 6

Part 2 Short Response/Grid In

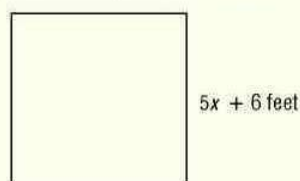
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. The diagram shows the dimensions of the cargo area of a delivery truck.



What is the maximum volume of cargo, in cubic feet, that can fit in the truck?
(Prerequisite Skill)

12. The perimeter of the square below is 204 feet. What is the value of x ? (Lesson 3-4)



13. What is the x -intercept of the graph of $4x + 3y = 12$? (Lesson 4-5)
14. What are the slope and the y -intercept of the graph of the equation $4x - 2y = 5$? (Lesson 5-4)
15. Solve the following system of equations.
(Lesson 7-2)

$$\begin{aligned} 5x - y &= 10 \\ 7x - 2y &= 11 \end{aligned}$$



Test-Taking Tip

Questions 11 and 12 To prepare for a standardized test, make flash cards of key mathematical terms, such as "perimeter" and "volume." Use the glossary of your textbook to determine the important terms and their correct definitions.

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- (A) the quantity in Column A is greater,
(B) the quantity in Column B is greater,
(C) the two quantities are equal, or
(D) the relationship cannot be determined from the information given.

Column A	Column B
16. 3^4	9^2

(Lesson 1-1)

17. the slope of the line that contains $A(2, 4)$ and $B(-1, 3)$	the slope of the line that contains $C(-2, 1)$ and $D(5, 3)$
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(Lesson 5-1)

18. $x - 3y = 11$ $3x + y = 13$	
y	0

(Lesson 7-4)

19. $3x - 2y = 19$ $5x + 4y = 17$	
x	y

(Lesson 7-4)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

20. The manager of a movie theater found that Saturday's sales were \$3675. He knew that a total of 650 tickets were sold Saturday. Adult tickets cost \$7.50, and children's tickets cost \$4.50. (Lesson 7-2)
- Write equations to represent the number of tickets sold and the amount of money collected.
 - How many of each kind of ticket were sold? Show your work. Include all steps.