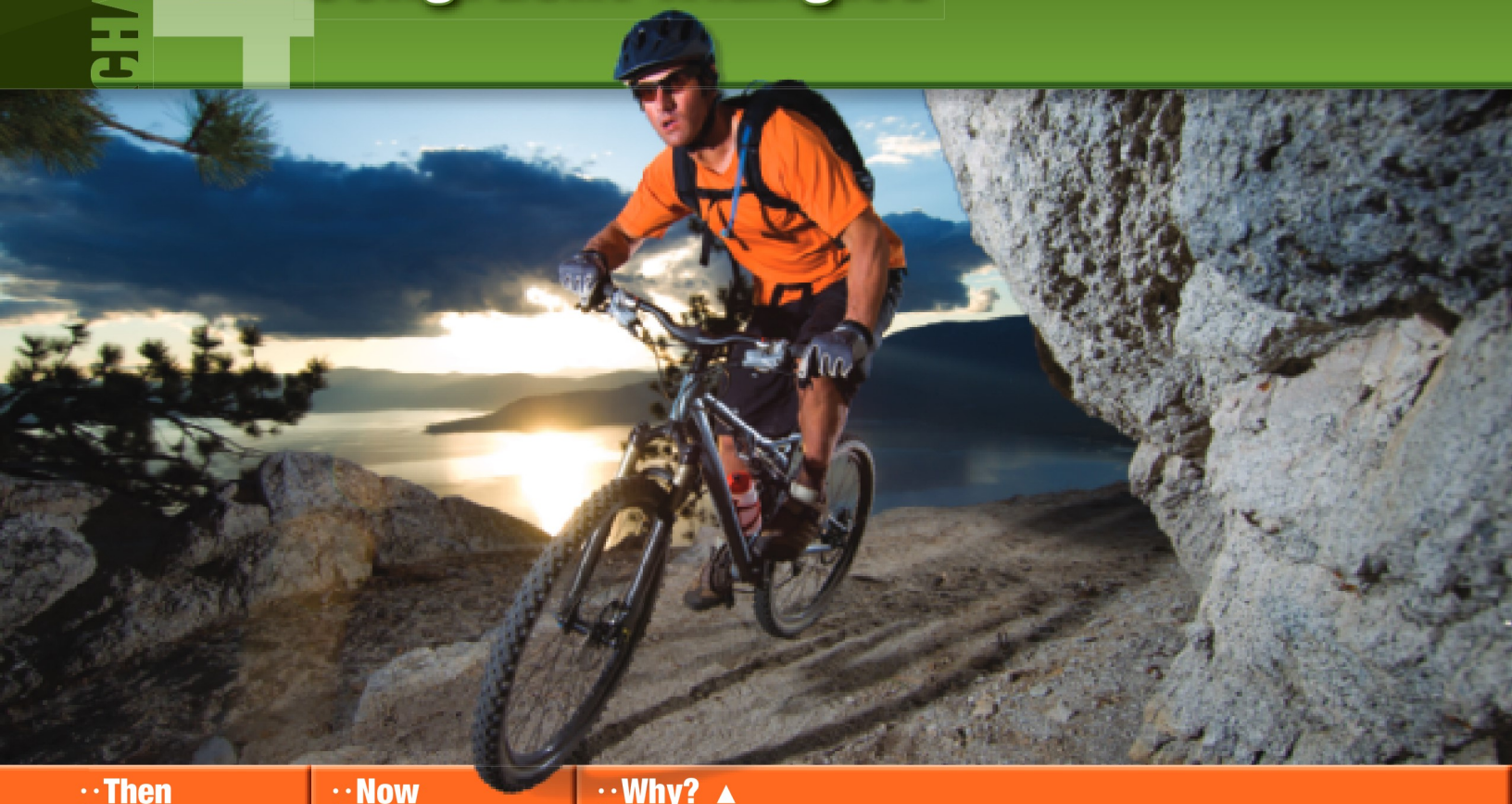
CHAPTER 4

Congruent Triangles



Then

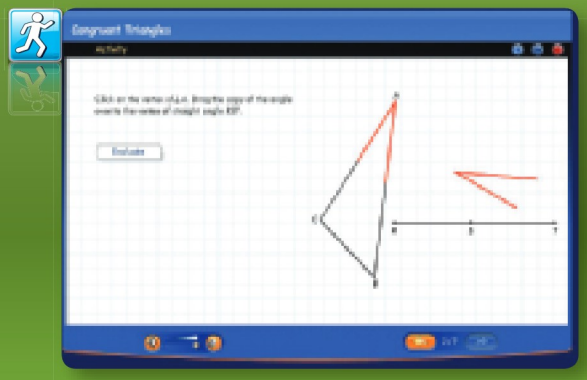
- You learned about segments, angles, and discovered relationships between their measures.

Now

- In this chapter, you will:
  - Apply special relationships about the interior and exterior angles of triangles.
  - Identify corresponding parts of congruent triangles and prove triangles congruent.
  - Learn about the special properties of isosceles and equilateral triangles

Why? ▲

- **FITNESS** Triangles are used to add strength to many structures, including fitness equipment such as bike frames.



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Scott Markewitz/Getty Images

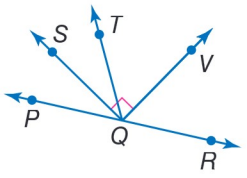
Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

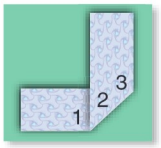
QuickCheck

Classify each angle as *right*, *acute*, or *obtuse*.

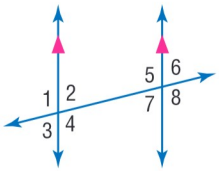


1. $m\angle VQS$
2. $m\angle TQV$
3. $m\angle PQV$

4. **ORIGAMI** The origami fold involves folding a strip of paper so that the lower edge of the strip forms a right angle with itself. Identify each angle as *right*, *acute*, or *obtuse*.



ALGEBRA Use the figure to find the indicated variable(s). Explain your reasoning.



5. Find x if $m\angle 3 = x - 12$ and $m\angle 6 = 72$.
6. If $m\angle 4 = 2y + 32$ and $m\angle 5 = 3y - 3$, find y.

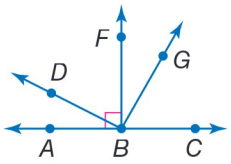
Find the distance between each pair of points.

7. $F(3, 6)$, $G(7, -4)$
8. $X(-2, 5)$, $Y(1, 11)$
9. $R(8, 0)$, $S(-9, 6)$
10. $A(14, -3)$, $B(9, -9)$
11. **MAPS** Miranda laid a coordinate grid on a map of a state where each 1 unit is equal to 10 miles. If her city is located at $(-8, -12)$ and the state capital is at $(0, 0)$, find the distance from her city to the capital to the nearest tenth of a mile.

QuickReview

Example 1

Classify each angle as *right*, *acute*, or *obtuse*.

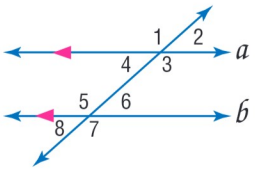


a. $m\angle ABG$
Point G on angle $\angle ABG$ lies on the exterior of right angle $\angle ABF$, so $\angle ABG$ is an obtuse angle.

b. $m\angle DBA$
Point D on angle $\angle DBA$ lies on the interior of right angle $\angle FBA$, so $\angle DBA$ is an acute angle.

Example 2

In the figure, $m\angle 4 = 42$. Find $m\angle 7$.



$\angle 7$ and $\angle 1$ are alternate interior angles, so they are congruent. $\angle 1$ and $\angle 4$ are a linear pair, so they are supplementary. Therefore, $\angle 7$ is supplementary to $\angle 1$. The measure of $\angle 7$ is $180 - 42$ or 138.

Example 3

Find the distance between $J(5, 2)$ and $K(11, -7)$.

$JK = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(11 - 5)^2 + [(-7) - 2]^2}$ Substitute.

$= \sqrt{6^2 + (-9)^2}$ Subtract.

$= \sqrt{36 + 81}$ or $\sqrt{117}$ Simplify.

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

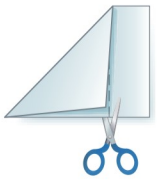
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 4. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

FOLDABLES StudyOrganizer

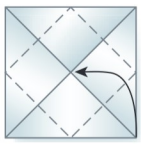
Congruent Triangles Make this Foldable to help you organize your Chapter 4 notes about congruent triangles. Begin with a sheet of $8\frac{1}{2}$" × 11" paper.

1 **Fold** into a taco forming a square. Cut off the excess paper strip formed by the square.



2 **Open** the fold and refold it the opposite way forming another taco and an X fold pattern.

3 **Open** and fold the corners toward the center point of the X forming a small square.

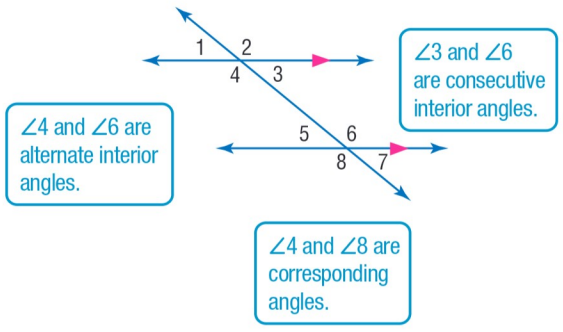


4 **Label** the flaps as shown.

NewVocabulary

English		Español
equiangular triangle	p. 237	triángulo equiangular
equilateral triangle	p. 238	triángulo equilátero
isosceles triangle	p. 238	triángulo isósceles
scalene triangle	p. 238	triángulo escaleno
auxiliary line	p. 246	linea auxiliar
congruent	p. 255	congruente
congruent polygons	p. 255	polígonos congruentes
corresponding parts	p. 255	partes correspondientes
included angle	p. 266	ángulo incluido
included side	p. 275	lado incluido
base angle	p. 285	ángulo de la base
transformation	p. 296	transformación
preimage	p. 296	preimagen
image	p. 296	imagen
reflection	p. 296	reflexión
translation	p. 296	traslación
rotation	p. 296	rotación

ReviewVocabulary



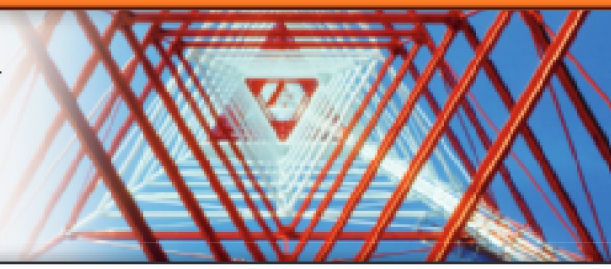
LESSON 4-1

Classifying Triangles

Then
- You measured and classified angles.

Now
1. Identify and classify triangles by angle measures.
2. Identify and classify triangles by side measures.

Why?
- Radio transmission towers are designed to support antennas for broadcasting radio or television signals. The structure of the tower shown reveals a pattern of triangular braces.



New Vocabulary
- acute triangle
- equiangular triangle
- obtuse triangle
- right triangle
- equilateral triangle
- isosceles triangle
- scalene triangle

Common Core State Standards

Content Standards

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

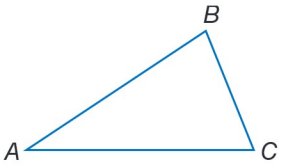
Mathematical Practices

2 Reason abstractly and quantitatively.

6 Attend to precision.

1 Classify Triangles by Angles

Recall that a triangle is a three-sided polygon. Triangle ABC, written $\triangle ABC$, has parts that are named using A, B, and C.



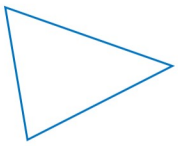
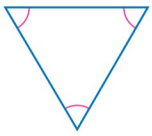
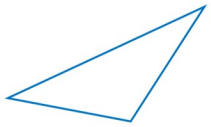
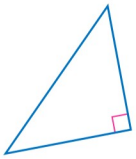
The sides of $\triangle ABC$ are $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

The vertices are points A, B, and C.

The angles are $\angle BAC$ or $\angle A$, $\angle ABC$ or $\angle B$, and $\angle BCA$ or $\angle C$.

Triangles can be classified in two ways—by their angles or by their sides. All triangles have at least two acute angles, but the third angle is used to classify the triangle.

KeyConcept Classifications of Triangles by Angles

acute triangle	equiangular triangle	obtuse triangle	right triangle
			
3 acute angles	3 congruent acute angles	1 obtuse angle	1 right angle

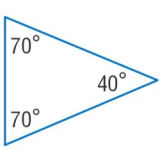
An equiangular triangle is a special kind of acute triangle.

When classifying triangles, be as specific as possible. While a triangle with three congruent acute angles is an acute triangle, it is more specific to classify it as an equiangular triangle.

Example 1 Classify Triangles by Angles

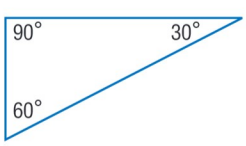
Classify each triangle as *acute, equiangular, obtuse,* or *right*.

a.



The triangle has three acute angles that are not all equal. It is an acute triangle.

b.



One angle of the triangle measures 90, so it is a right angle. Since the triangle has a right angle, it is a right triangle.

ReviewVocabulary

acute angle an angle with a degree measure less than 90

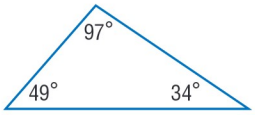
right angle an angle with a degree measure of 90

obtuse angle an angle with a degree measure greater than 90

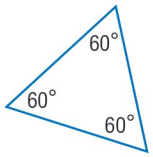
GuidedPractice

Classify each triangle as *acute, equiangular, obtuse,* or *right.*

1A.



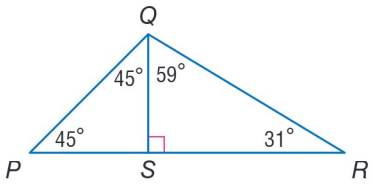
1B.



Example 2 Classify Triangles by Angles Within Figures

Classify $\triangle PQR$ as *acute, equiangular, obtuse,* or *right*. Explain your reasoning.

Point S is in the interior of $\angle PQR$, so by the Angle Addition Postulate, $m\angle PQR = m\angle PQS + m\angle SQR$. By substitution, $m\angle PQR = 45 + 59$ or 104.



Since $\triangle PQR$ has one obtuse angle, it is an obtuse triangle.

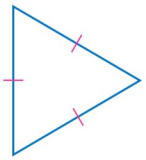
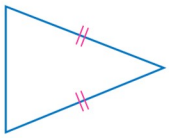
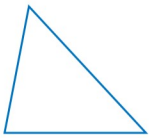
GuidedPractice

2. Use the diagram to classify $\triangle PQS$ as *acute, equiangular, obtuse* or *right*. Explain your reasoning.

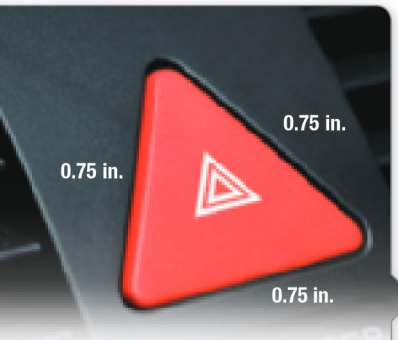
2 Classify Triangles by Sides

Triangles can also be classified according to the number of congruent sides they have. To indicate that sides of a triangle are congruent, an equal number of hash marks is drawn on the corresponding sides.

KeyConcept Classifications of Triangles by Sides

equilateral triangle	isosceles triangle	scalene triangle
		
3 congruent sides	at least 2 congruent sides	no congruent sides

An equilateral triangle is a special kind of isosceles triangle.



Real-WorldLink

In many cars, hazard lights are activated by pushing a small button located near the steering column. The switch is usually an icon shaped like an equilateral triangle.

Source: General Motors

Real-World Example 3 Classify Triangles by Sides

MUSIC Classify the sound box of the Russian lute below as *equilateral, isosceles,* or *scalene*.

Two sides have the same measure, 16 inches, so the triangle has two congruent sides. The triangle is isosceles.

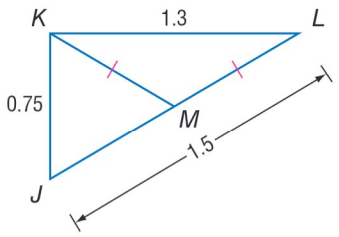
GuidedPractice

3. DRIVING SAFETY Classify the button in the picture at the left by its sides.

(l)IfStop/SuperStock, (r)Stockbyte/Getty Images

Example 4 Classify Triangles by Sides Within Figures

If point M is the midpoint of $\overline{JL}$, classify $\triangle JKM$ as *equilateral*, *isosceles*, or *scalene*. Explain your reasoning.



By the definition of midpoint, $JM = ML$.

$JM + ML = JL$	Segment Addition Postulate
$ML + ML = 1.5$	Substitution
$2ML = 1.5$	Simplify.
$ML = 0.75$	Divide each side by 2.

$JM = ML$ or 0.75. Since $\overline{KM} \cong \overline{ML}$, $KM = ML$ or 0.75.

Since $KJ = JM = KM = 0.75$, the triangle has three sides with the same measure. Therefore, the triangle has three congruent sides, so it is equilateral.

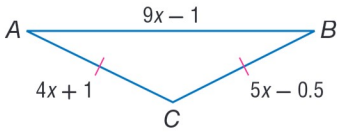
GuidedPractice

4. Classify $\triangle KML$ as *equilateral*, *isosceles*, or *scalene*. Explain your reasoning.

You can also use the properties of isosceles and equilateral triangles to find missing values.

Example 5 Finding Missing Values

ALGEBRA Find the measures of the sides of isosceles triangle ABC.



Step 1 Find x.

$AC = CB$	Given
$4x + 1 = 5x - 0.5$	Substitution
$1 = x - 0.5$	Subtract $4x$ from each side.
$1.5 = x$	Add 0.5 to each side.

Step 2 Substitute to find the length of each side.

$AC = 4x + 1$	Given
$= 4(1.5) + 1$ or 7	$x = 1.5$
$CB = AC$	Given
$= 7$	$AC = 7$
$AB = 9x - 1$	Given
$= 9(1.5) - 1$	$x = 1.5$
$= 12.5$	Simplify.

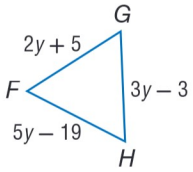
StudyTip

CCSS Perseverance In Example 5, to check your answer, test to see if $CB = AC$ when 1.5 is substituted for x in the expression for CB, $5x - 0.5$.

$CB = 5x - 0.5$
$= 5(1.5) - 0.5$ or 7 ✔

GuidedPractice

5. Find the measures of the sides of equilateral triangle FGH.

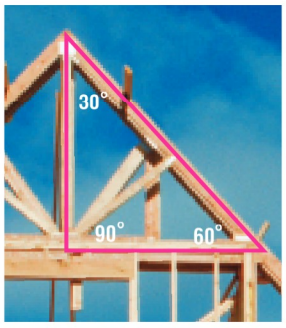


Check Your Understanding

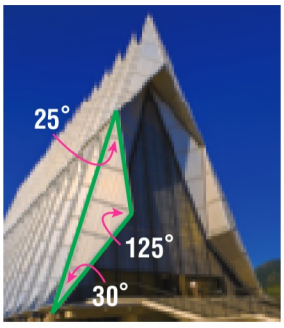
= Step-by-Step Solutions begin on page R14.

Example 1 **ARCHITECTURE** Classify each triangle as *acute, equiangular, obtuse,* or *right.*

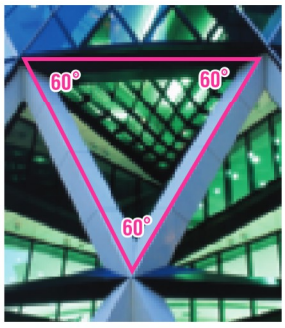
1.



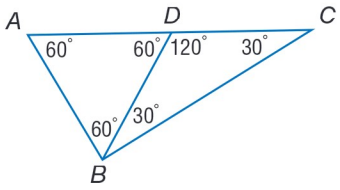
2.



3.



Example 2 Classify each triangle as *acute, equiangular, obtuse,* or *right.* Explain your reasoning.

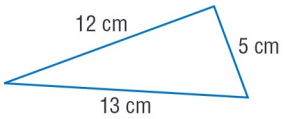


4. $\triangle ABD$
5. $\triangle BDC$
6. $\triangle ABC$

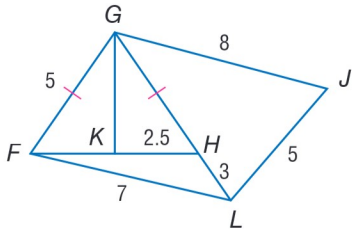
Example 3 CCSS **PRECISION** Classify each triangle as *equilateral, isosceles,* or *scalene.*

7.

8.



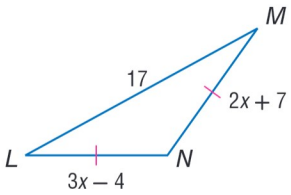
Example 4 If point K is the midpoint of $\overline{FH}$, classify each triangle in the figure at the right as *equilateral, isosceles,* or *scalene.*



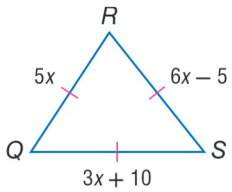
9. $\triangle FGH$
10. $\triangle GJL$
11. $\triangle FHL$

Example 5 **ALGEBRA** Find x and the measures of the unknown sides of each triangle.

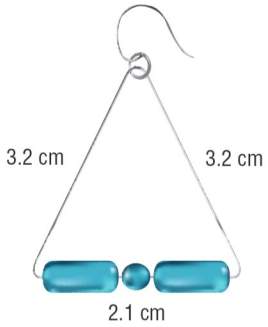
12.



13.



14. **JEWELRY** Suppose you are bending stainless steel wire to make the earring shown. The triangular portion of the earring is an isosceles triangle. If 1.5 centimeters are needed to make the hook portion of the earring, how many earrings can be made from 45 centimeters of wire? Explain your reasoning.

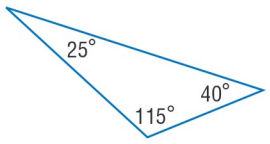


Practice and Problem Solving

Extra Practice is on page R4.

Example 1 Classify each triangle as *acute, equiangular, obtuse,* or *right.*

15.



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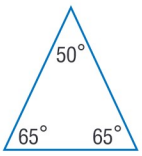

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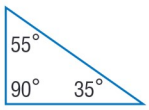

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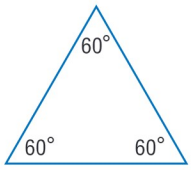

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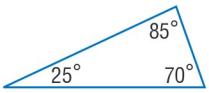

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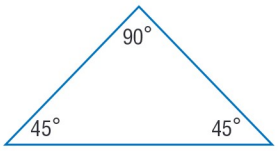


Example 2 CCSS **PRECISION** Classify each triangle as *acute, equiangular, obtuse,* or *right.*

21. $\triangle UYZ$

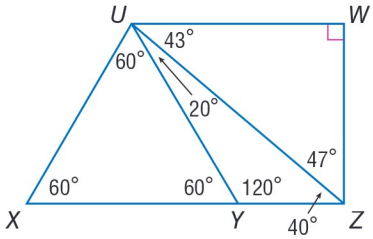
22. $\triangle BCD$

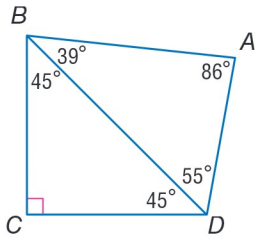
23. $\triangle ADB$

24. $\triangle UXZ$

25. $\triangle UWZ$

26. $\triangle UXY$



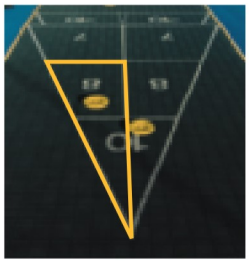


Example 3 Classify each triangle as *equilateral, isosceles,* or *scalene.*

27.

28.

29.



Example 4 If point C is the midpoint of $\overline{BD}$ and point E is the midpoint of $\overline{DF}$, classify each triangle as *equilateral, isosceles,* or *scalene.*

30. $\triangle ABC$

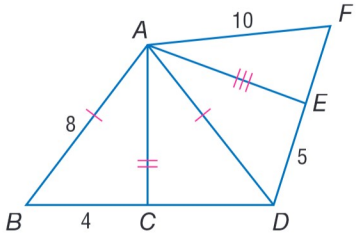
31. $\triangle AEF$

32. $\triangle ADF$

33. $\triangle ACD$

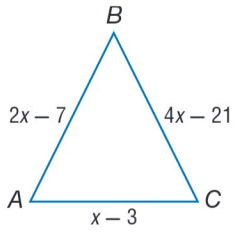
34. $\triangle AED$

35. $\triangle ABD$

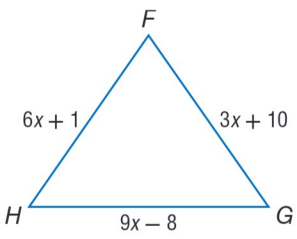


Example 5

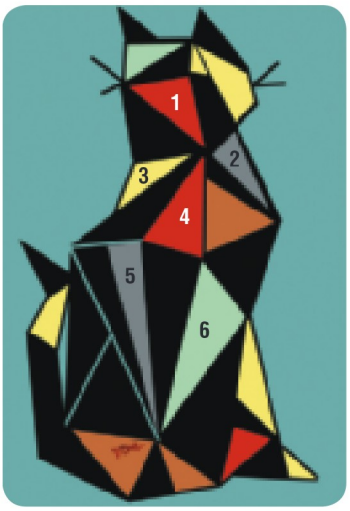
36. **ALGEBRA** Find x and the length of each side if $\triangle ABC$ is an isosceles triangle with $\overline{AB} \cong \overline{BC}$.



37. **ALGEBRA** Find x and the length of each side if $\triangle FGH$ is an equilateral triangle.



38. GRAPHIC ART Refer to the illustration shown. Classify each numbered triangle in *Kat* by its angles and by its sides. Use the corner of a sheet of notebook paper to classify angle measures and a ruler to measure sides.



Kat, 2002, by Diana Ong, computer graphic

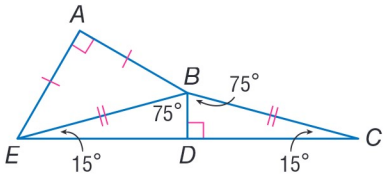
39. KALEIDOSCOPE Josh is building a kaleidoscope using PVC pipe, cardboard, bits of colored paper, and a 12-inch square mirror tile. The mirror tile is to be cut into strips and arranged to form an open prism with a base like that of an equilateral triangle. Make a sketch of the prism, giving its dimensions. Explain your reasoning.

CCSS PRECISION **Classify each triangle in the figure by its angles and sides.**

40. $\triangle ABE$

41. $\triangle EBC$

42. $\triangle BDC$



COORDINATE GEOMETRY **Find the measures of the sides of $\triangle XYZ$ and classify each triangle by its sides.**

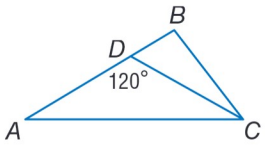
43. $X(-5, 9)$, $Y(2, 1)$, $Z(-8, 3)$

44. $X(7, 6)$, $Y(5, 1)$, $Z(9, 1)$

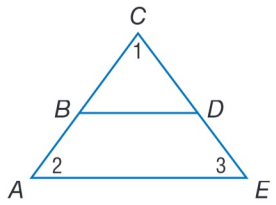
45. $X(3, -2)$, $Y(1, -4)$, $Z(3, -4)$

46. $X(-4, -2)$, $Y(-3, 7)$, $Z(4, -2)$

47. PROOF Write a paragraph proof to prove that $\triangle DBC$ is an acute triangle if $m\angle ADC = 120$ and $\triangle ABC$ is acute.



48. PROOF Write a two-column proof to prove that $\triangle BCD$ is equiangular if $\triangle ACE$ is equiangular and $\overline{BD} \parallel \overline{AE}$.



ALGEBRA **For each triangle, find x and the measure of each side.**

49. $\triangle FGH$ is an equilateral triangle with $FG = 3x - 10$, $GH = 2x + 5$, and $HF = x + 20$.

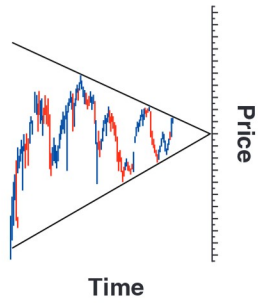
50. $\triangle JKL$ is isosceles with $\overline{JK} \cong \overline{KL}$, $JK = 4x - 1$, $KL = 2x + 5$, and $LJ = 2x - 1$.

51. $\triangle MNP$ is isosceles with $\overline{MN} \cong \overline{NP}$. MN is two less than five times x, NP is seven more than two times x, and PM is two more than three times x.

52. $\triangle RST$ is equilateral. RS is three more than four times x, ST is seven more than two times x, and TR is one more than five times x.

53. CONSTRUCTION Construct an equilateral triangle. Verify your construction using measurement and justify it using mathematics. (*Hint:* Use the construction for copying a segment.)

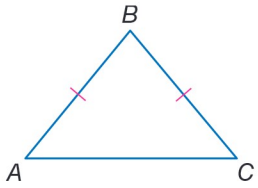
54. STOCKS Technical analysts use charts to identify patterns that can suggest future activity in stock prices. Symmetrical triangle charts are most useful when the fluctuation in the price of a stock is decreasing over time.



a. Classify by its sides and angles the triangle formed if a vertical line is drawn at any point on the graph.

b. How would the price have to fluctuate in order for the data to form an obtuse triangle? Draw an example to support your reasoning.

55. MULTIPLE REPRESENTATIONS In the diagram, the vertex *opposite* side $\overline{BC}$ is $\angle A$.



a. Geometric Draw four isosceles triangles, including one acute, one right, and one obtuse isosceles triangle. Label the vertices opposite the congruent sides as A and C. Label the remaining vertex B. Then measure the angles of each triangle and label each angle with its measure.

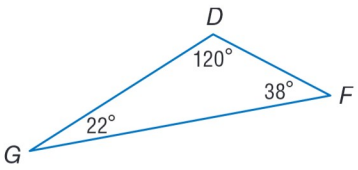
b. Tabular Measure all the angles of each triangle. Organize the measures for each triangle into a table. Include a column in your table to record the sum of these measures.

c. Verbal Make a conjecture about the measures of the angles that are opposite the congruent sides of an isosceles triangle. Then make a conjecture about the sum of the measures of the angles of an isosceles triangle.

d. Algebraic If x is the measure of one of the angles opposite one of the congruent sides in an isosceles triangle, write expressions for the measures of each of the other two angles in the triangle. Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

56. ERROR ANALYSIS Elaina says that $\triangle DFG$ is obtuse. Ines disagrees, explaining that the triangle has more acute angles than obtuse angles so it must be acute. Is either of them correct? Explain your reasoning.



CCSS PRECISION **Determine whether the statements below are *sometimes*, *always*, or *never* true. Explain your reasoning.**

57. Equiangular triangles are also right triangles.

58. Equilateral triangles are isosceles.

59. Right triangles are equilateral.

60. CHALLENGE An equilateral triangle has sides that measure $5x + 3$ units and $7x - 5$ units. What is the perimeter of the triangle? Explain.

OPEN ENDED **Draw an example of each type of triangle below using a protractor and a ruler. Label the sides and angles of each triangle with their measures. If not possible, explain why not.**

61. scalene right

62. isosceles obtuse

63. equilateral obtuse

64. WRITING IN MATH Explain why classifying an equiangular triangle as an *acute* equiangular triangle is unnecessary.

Standardized Test Practice

65. Which type of triangle can serve as a counterexample to the conjecture below?

> If two angles of a triangle are acute, then the measure of the third angle must be greater than or equal to 90.

A equilateral **C** right
B obtuse **D** scalene

66. ALGEBRA A baseball glove originally cost $84.50. Kenji bought it at 40% off. How much was deducted from the original price?

F $50.70 **H** $33.80
G $44.50 **J** $32.62

67. GRIDDED RESPONSE Jorge is training for a 20-mile race. Jorge runs 7 miles on Monday, Tuesday, and Friday, and 12 miles on Wednesday and Saturday. After 6 weeks of training, Jorge will have run the equivalent of how many races?

68. SAT/ACT What is the slope of the line determined by the equation $2x + y = 5$?

A $-\frac{5}{2}$ **D** 2
B -2 **E** $\frac{5}{2}$
C -1

Spiral Review

Find the distance between each pair of parallel lines with the given equations. (Lesson 3-6)

69. $x = -2$, $x = 5$

70. $y = -6$, $y = 1$

71. $y = 2x + 3$, $y = 2x - 7$

72. $y = x + 2$, $y = x - 4$

73. FOOTBALL When striping the practice football field, Mr. Hawkins first painted the sidelines. Next he marked off 10-yard increments on one sideline. He then constructed lines perpendicular to the sidelines at each 10-yard mark. Why does this guarantee that the 10-yard lines will be parallel? (Lesson 3-5)

Identify the hypothesis and conclusion of each conditional statement. (Lesson 2-3)

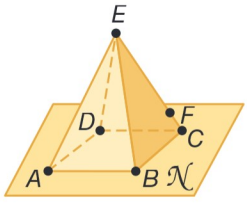
74. If three points lie on a line, then they are collinear.

75. If you are a teenager, then you are at least 13 years old.

76. If $2x + 6 = 10$, then $x = 2$.

77. If you have a driver's license, then you are at least 16 years old.

Refer to the figure at the right. (Lesson 1-1)



78. How many planes appear in this figure?

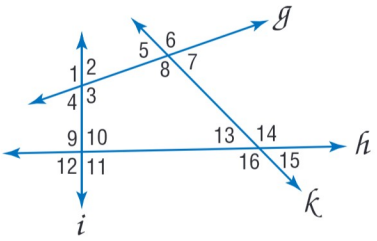
79. Name the intersection of plane AEB with plane $\mathcal{N}$.

80. Name three points that are collinear.

81. Are points D, E, C, and B coplanar?

Skills Review

Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior angles*.



82. $\angle 5$ and $\angle 3$

83. $\angle 9$ and $\angle 4$

84. $\angle 11$ and $\angle 13$

85. $\angle 1$ and $\angle 11$

EXPLORE

4-2 Geometry Lab
Angles of Triangles

In this lab, you will find special relationships among the angles of a triangle.

Common Core State Standards
Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
Mathematical Practices 5

Activity 1 Interior Angles of a Triangle

Step 1

Draw and cut out several different triangles. Label the vertices A, B, and C.

Step 2

For each triangle, fold vertex B down so that the fold line is parallel to $\overline{AC}$. Relabel as vertex B.

Step 3

Then fold vertices A and C so that they meet vertex B. Relabel as vertices A and C.

Analyze the Results

1. Angles A, B, and C are called *interior angles* of triangle ABC. What type of figure do these three angles form when joined together in Step 3?
2. **Make a conjecture** about the sum of the measures of the interior angles of a triangle.

Activity 2 Exterior Angles of a Triangle

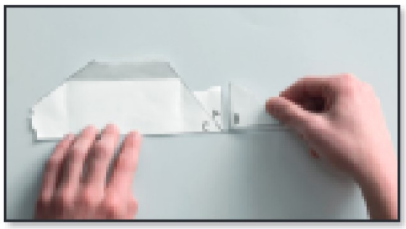
Step 1

Unfold each triangle from Activity 1 and place each on a separate piece of paper. Extend $\overline{AC}$ as shown.

Step 2

For each triangle, tear off $\angle A$ and $\angle B$.

Step 3



Arrange $\angle A$ and $\angle B$ so that they fill the angle adjacent to $\angle C$ as shown.

Model and Analyze the Results

3. The angle adjacent to $\angle C$ is called an *exterior angle* of triangle ABC. **Make a conjecture** about the relationship among $\angle A$, $\angle B$, and the exterior angle at C.
4. Repeat the steps in Activity 2 for the exterior angles of $\angle A$ and $\angle B$ in each triangle.
5. **Make a conjecture** about the measure of an exterior angle and the sum of the measures of its nonadjacent interior angles.

Ed-Imaging

LESSON 4-2

Angles of Triangles

Then

- You classified triangles by their side or angle measures.

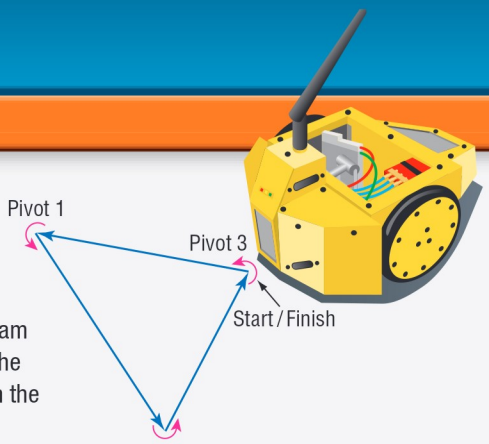
Now

1. Apply the Triangle Angle-Sum Theorem.
2. Apply Exterior Angle Theorem.

Why?

- Massachusetts Institute of Technology (MIT) sponsors the annual *Design 2.007* contest in which students design and build a robot.

One test of a robot's movements is to program it to move in a triangular path. The sum of the measures of the pivot angles through which the robot must turn will always be the same.



New Vocabulary

auxiliary line
exterior angle
remote interior angles
flow proof
corollary

Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

3 Construct viable arguments and critique the reasoning of others.

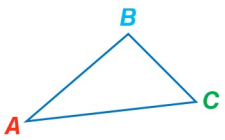
1 Triangle Angle-Sum Theorem

The Triangle Angle-Sum Theorem gives the relationship among the interior angle measures of any triangle.

Theorem 4.1 Triangle Angle-Sum Theorem

Words The sum of the measures of the angles of a triangle is 180.

Example $m\angle A + m\angle B + m\angle C = 180$



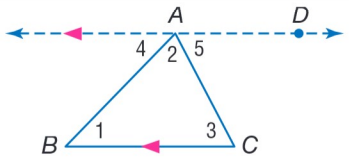
The proof of the Triangle Angle-Sum Theorem requires the use of an auxiliary line. An **auxiliary line** is an extra line or segment drawn in a figure to help analyze geometric relationships. As with any statement in a proof, you must justify any properties of an auxiliary line that you have drawn.

Proof Triangle Angle-Sum Theorem

Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$

Proof:



Statements	Reasons
1. $\triangle ABC$	1. Given
2. Draw $\overleftrightarrow{AD}$ through A parallel to $\overline{BC}$.	2. Parallel Postulate
3. $\angle 4$ and $\angle BAD$ form a linear pair.	3. Def. of a linear pair
4. $\angle 4$ and $\angle BAD$ are supplementary.	4. If 2 $\measuredangle$ form a linear pair, they are supplementary.
5. $m\angle 4 + m\angle BAD = 180$	5. Def. of suppl. $\measuredangle$
6. $m\angle BAD = m\angle 2 + m\angle 5$	6. Angle Addition Postulate
7. $m\angle 4 + m\angle 2 + m\angle 5 = 180$	7. Substitution
8. $\angle 4 \cong \angle 1, \angle 5 \cong \angle 3$	8. Alt. Int. $\measuredangle$ Theorem
9. $m\angle 4 = m\angle 1, m\angle 5 = m\angle 3$	9. Def. of $\cong$ $\measuredangle$
10. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	10. Substitution

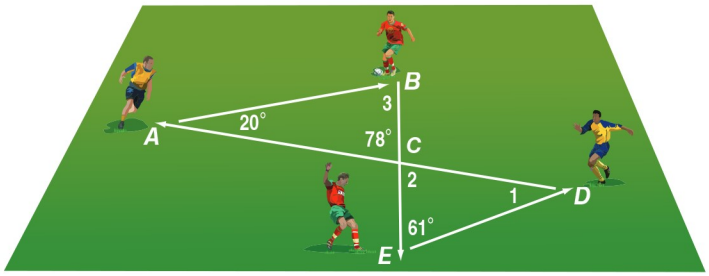
Real-World Link

The pass-and-move soccer drill incorporates several fundamental aspects of passing. All passes in this drill are made in a triangle, which is the basis of all ball movement. Additionally, the players are forced to move immediately after passing the ball.

The Triangle Angle-Sum Theorem can be used to determine the measure of the third angle of a triangle when the other two angle measures are known.

Real-World Example 1 Use the Triangle Angle-Sum Theorem

SOCCER **The diagram shows the path of the ball in a passing drill created by four friends. Find the measure of each numbered angle.**



Understand Examine the information given in the diagram. You know the measures of two angles of one triangle and only one measure of another. You also know that $\angle ACB$ and $\angle 2$ are vertical angles.

Plan Find $m\angle 3$ using the Triangle Angle-Sum Theorem, because the measures of two angles of $\angle ABC$ are known. Use the Vertical Angles Theorem to find $m\angle 2$. Then you will have enough information to find the measure of $\angle 1$ in $\triangle CDE$.

Solve

$m\angle 3 + m\angle BAC + m\angle ACB = 180$ Triangle Angle-Sum Theorem

$m\angle 3 + 20 + 78 = 180$ Substitution

$m\angle 3 + 98 = 180$ Simplify.

$m\angle 3 = 82$ Subtract 98 from each side.

$\angle ACB$ and $\angle 2$ are congruent vertical angles. So, $m\angle 2 = 78$.

Use $m\angle 2$ and $\angle CED$ of $\triangle CDE$ to find $m\angle 1$.

$m\angle 1 + m\angle 2 + m\angle CED = 180$ Triangle Angle-Sum Theorem

$m\angle 1 + 78 + 61 = 180$ Substitution

$m\angle 1 + 139 = 180$ Simplify.

$m\angle 1 = 41$ Subtract 139 from each side.

Check The sums of the measures of the angles of $\triangle ABC$ and $\triangle CDE$ should be 180.

$\triangle ABC$: $m\angle 3 + m\angle BAC + m\angle ACB = 82 + 20 + 78$ or 180 ✓

$\triangle CDE$: $m\angle 1 + m\angle 2 + m\angle CED = 41 + 78 + 61$ or 180 ✓

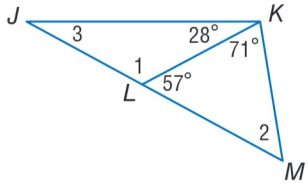
Problem-Solving Tip

CCSS Sense-Making Often a complex problem can be more easily solved if you first break it into more manageable parts. In Example 1, before you can find $m\angle 1$, you must first find $m\angle 2$.

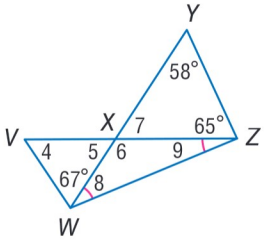
Guided Practice

Find the measures of each numbered angle.

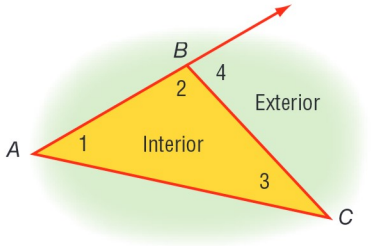
1A.



1B.



2 Exterior Angle Theorem In addition to its three interior angles, a triangle can have **exterior angles** formed by one side of the triangle and the extension of an adjacent side. Each exterior angle of a triangle has two **remote interior angles** that are not adjacent to the exterior angle.

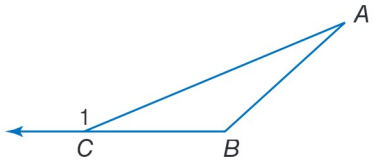


$\angle 4$ is an exterior angle of $\triangle ABC$. Its two remote interior angles are $\angle 1$ and $\angle 3$.

Theorem 4.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Example $m\angle A + m\angle B = m\angle 1$



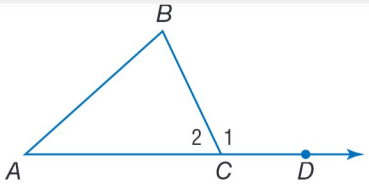
ReadingMath
Flowchart Proof A flow proof is sometimes called a *flowchart* proof.

A **flow proof** uses statements written in boxes and arrows to show the logical progression of an argument. The reason justifying each statement is written below the box. You can use a flow proof to prove the Exterior Angle Theorem.

Proof Exterior Angle Theorem

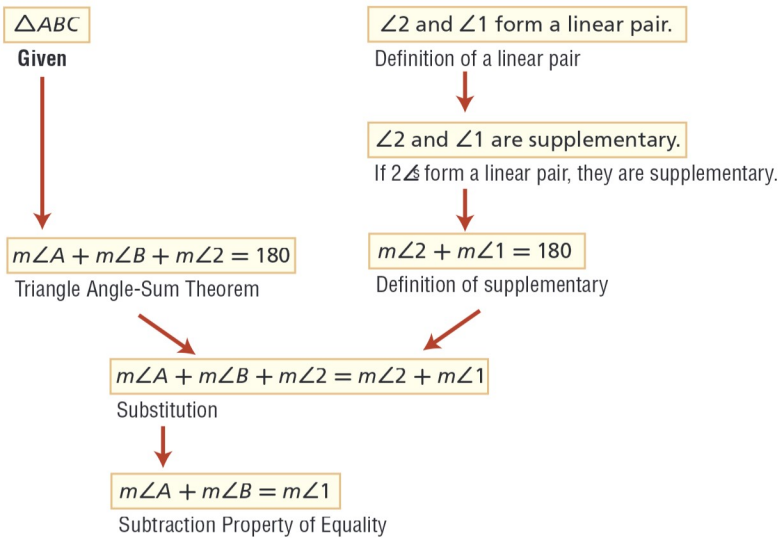
Given: $\triangle ABC$

Prove: $m\angle A + m\angle B = m\angle 1$



StudyTip
Flow Proofs Flow proofs can be written vertically or horizontally.

Flow Proof:



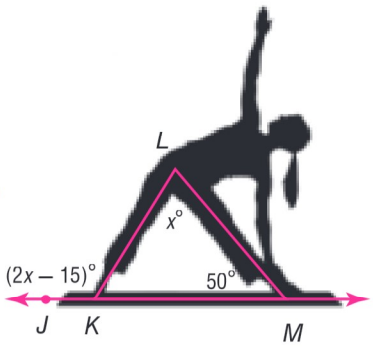
The Exterior Angle Theorem can also be used to find missing measures.

Real-WorldCareer

Personal Trainer Personal trainers instruct and motivate individuals in exercise activities. They demonstrate various exercises and help clients improve their exercise techniques. Personal trainers must obtain certification in the fitness field.

Real-World Example 2 Use the Exterior Angle Theorem

FITNESS Find the measure of $\angle JKL$ in the Triangle Pose shown.



$m\angle KLM + m\angle LMK = m\angle JKL$ Exterior Angle Theorem

$x + 50 = 2x - 15$ Substitution

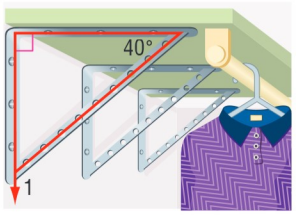
$50 = x - 15$ Subtract x from each side.

$65 = x$ Add 15 to each side.

So, $m\angle JKL = 2(65) - 15$ or 115.

GuidedPractice

2. CLOSET ORGANIZING Tanya mounts the shelving bracket shown to the wall of her closet. What is the measure of $\angle 1$, the angle that the bracket makes with the wall?



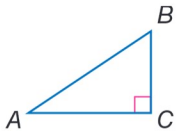
A **corollary** is a theorem with a proof that follows as a direct result of another theorem. As with a theorem, a corollary can be used as a reason in a proof. The corollaries below follow directly from the Triangle Angle-Sum Theorem.

Corollaries Triangle Angle-Sum Corollaries

4.1 The acute angles of a right triangle are complementary.

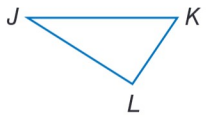
Abbreviation: *Acute ∡s of a rt. △ are comp.*

Example: If $\angle C$ is a right angle, then $\angle A$ and $\angle B$ are complementary.



4.2 There can be at most one right or obtuse angle in a triangle.

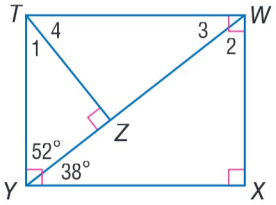
Example: If $\angle L$ is a right or an obtuse angle, then $\angle J$ and $\angle K$ must be acute angles.



You will prove Corollaries 4.1 and 4.2 in Exercises 34 and 35.

Example 3 Find Angle Measures in Right Triangles

Find the measures of each numbered angle.



$m\angle 1 + m\angle TYZ = 90$ Acute ∡s of a rt. △ are comp.

$m\angle 1 + 52 = 90$ Substitution

$m\angle 1 = 38$ Subtract 52 from each side.

GuidedPractice

3A. $\angle 2$ **3B.** $\angle 3$ **3C.** $\angle 4$

StudyTip

Check for Reasonableness
When you are solving for the measure of one or more angles of a triangle, always check to make sure that the sum of the angle measures is 180.

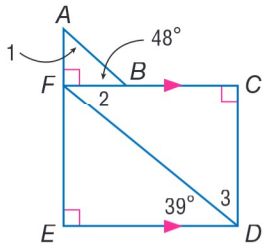
Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 Find the measures of each numbered angle.

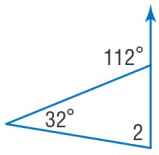
1.

2.

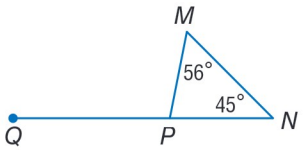


Example 2 Find each measure.

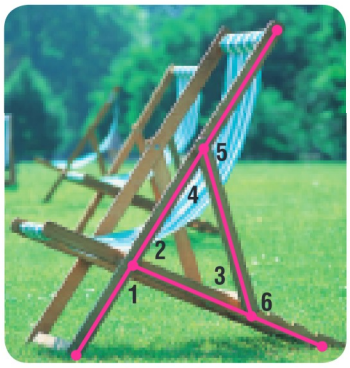
3. $m\angle 2$



4. $m\angle MPQ$



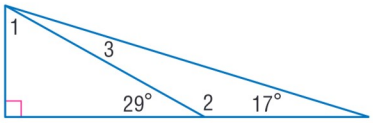
DECK CHAIRS The brace of this deck chair forms a triangle with the rest of the chair's frame as shown. If $m\angle 1 = 102$ and $m\angle 3 = 53$, find each measure.



5. $m\angle 4$
6. $m\angle 6$
7. $m\angle 2$
8. $m\angle 5$

Example 3 CCSS **REGULARITY** Find each measure.

9. $m\angle 1$
10. $m\angle 3$
11. $m\angle 2$



Practice and Problem Solving

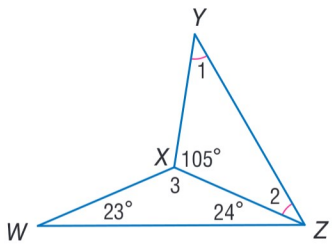
Extra Practice is on page R4.

Example 1 Find the measure of each numbered angle.

12.

13.

14.



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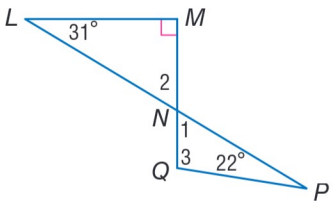
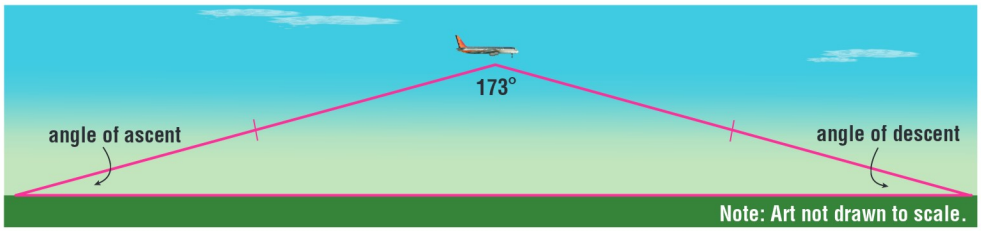


16. AIRPLANES The path of an airplane can be modeled using two sides of a triangle as shown. The distance covered during the plane's ascent is equal to the distance covered during its descent.

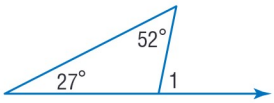


a. Classify the model using its sides and angles.

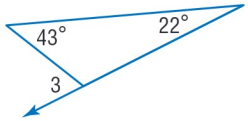
b. The angles of ascent and descent are congruent. Find their measures.

Example 2 **Find each measure.**

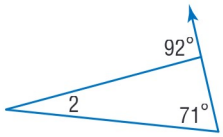
17. $m\angle 1$



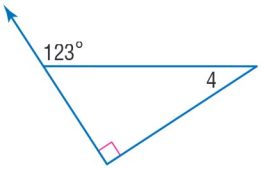
18. $m\angle 3$



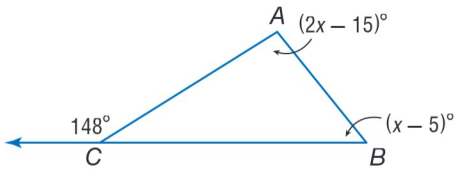
19. $m\angle 2$



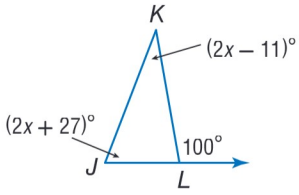
20. $m\angle 4$



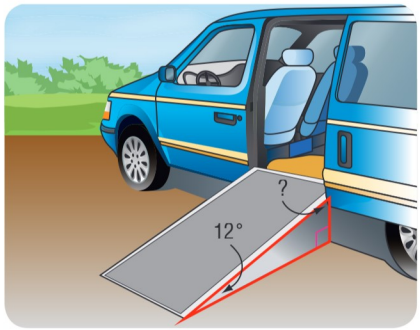
21 $m\angle ABC$



22. $m\angle JKL$



Example 3 **23. WHEELCHAIR RAMP** Suppose the wheelchair ramp shown makes a 12° angle with the ground. What is the measure of the angle the ramp makes with the van door?



CCSS **REGULARITY** **Find each measure.**

24. $m\angle 1$ **25.** $m\angle 2$

26. $m\angle 3$ **27.** $m\angle 4$

28. $m\angle 5$ **29.** $m\angle 6$