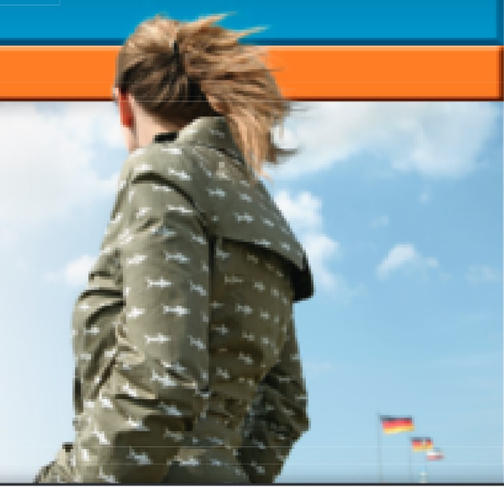
LESSON 4-7

Congruence Transformations



Then

- You proved whether two triangles were congruent.

Now

1. Identify reflections, translations, and rotations.
2. Verify congruence after a congruence transformation.

Why?

- The fashion industry often uses prints that display patterns. Many of these patterns are created by taking one figure and sliding it to create another figure in a different location, flipping the figure to create a mirror image of the original, or turning the original figure to create a new one.

New Vocabulary

transformation
preimage
image
congruence transformation
isometry
reflection
translation
rotation

Common Core State Standards

Content Standards

G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Mathematical Practices

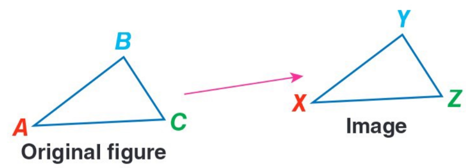
1 Make sense of problems and persevere in solving them.

7 Look for and make use of structure.

1 Identify Congruence Transformations

A **transformation** is an operation that maps an original geometric figure, the **preimage**, onto a new figure called the **image**. A transformation can change the position, size, or shape of a figure.

A transformation can be noted using an arrow. The transformation statement $\triangle ABC \rightarrow \triangle XYZ$ tells you that A is mapped to X, B is mapped to Y, and C is mapped to Z.

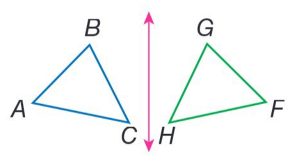


A **congruence transformation**, also called a *rigid transformation* or an **isometry**, is one in which the position of the image may differ from that of the preimage, but the two figures remain congruent. The three main types of congruence transformations are shown below.

KeyConcept Reflections, Translations, and Rotations

A **reflection** or *flip* is a transformation over a line called the *line of reflection*. Each point of the preimage and its image are the same distance from the line of reflection.

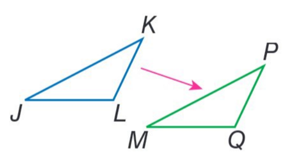
Example



$\triangle ABC \longrightarrow \triangle FGH$

A **translation** or *slide* is a transformation that moves all points of the original figure the same distance in the same direction.

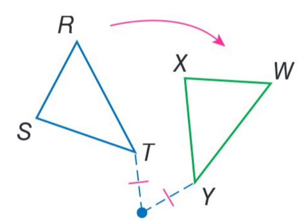
Example



$\triangle JKL \longrightarrow \triangle MPQ$

A **rotation** or *turn* is a transformation around a fixed point called the *center of rotation*, through a specific angle, and in a specific direction. Each point of the original figure and its image are the same distance from the center.

Example



$\triangle RST \longrightarrow \triangle WXY$

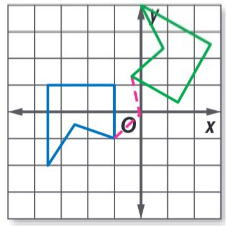
StudyTip

Transformations Not all transformations preserve congruence. Only transformations that do not change the size or shape of the figure are congruence transformations. You will learn more about transformations in Chapter 9.

Example 1 Identify Congruence Transformations

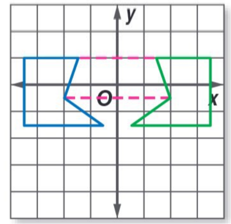
Identify the type of congruence transformation shown as a *reflection, translation,* or *rotation.*

a.



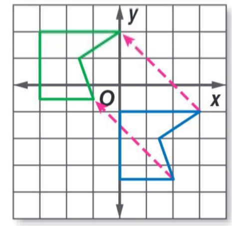
Each vertex and its image are the same distance from the origin. The angles formed by each pair of corresponding points and the origin are congruent. This is a rotation.

b.



Each vertex and its image are the same distance from the *y*-axis. This is a reflection.

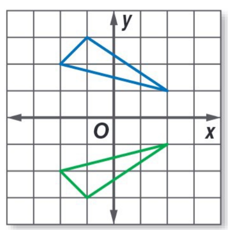
c.



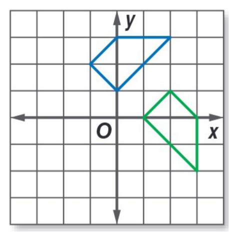
Each vertex and its image are in the same position, just 3 units left and 3 units up. This is a translation.

GuidedPractice

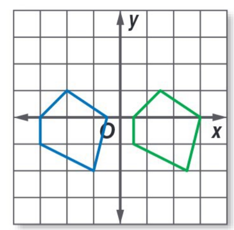
1A.



1B.



1C.



Some real-world motions or objects can be represented by transformations.

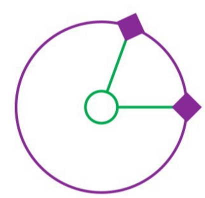
Real-WorldLink

The game shown above involves a weight attached to a ring that you can place around your ankle. As the rope passes in front of your other foot, you skip over it.

Corbis/SuperStock

Real-World Example 2 Identify a Real-World Transformation

GAMES Refer to the information at the left. Identify the type of congruence transformation shown in the diagram as a *reflection, translation,* or *rotation.*

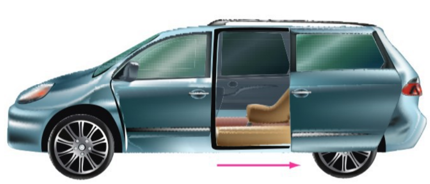


The position of the weight at different times is an example of a rotation. The center of rotation is the person's ankle.

GuidedPractice

Identify the type of congruence transformation shown as a *reflection, translation,* or *rotation.*

2A.



2B.

2 Verify Congruence

You can verify that reflections, translations, and rotations of triangles produce congruent triangles using SSS.

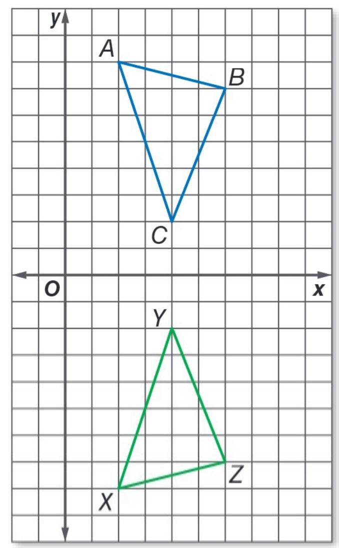
Example 3 Verify Congruence after a Transformation

Triangle XZY with vertices $X(2, -8)$, $Z(6, -7)$, and $Y(4, -2)$ is a transformation of $\triangle ABC$ with vertices $A(2, 8)$, $B(6, 7)$, and $C(4, 2)$. Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.

Understand You are asked to identify the type of transformation—reflection, translation, or rotation. Then, you need to show that the two figures are congruent.

Plan Use the Distance Formula to find the measure of each side. Then show that the two triangles are congruent by SSS.

Solve Graph each figure. The transformation appears to be a reflection over the x-axis. Find the measures of the sides of each triangle.



$$AB = \sqrt{(6-2)^2 + (7-8)^2} \text{ or } \sqrt{17}$$

$$BC = \sqrt{(6-4)^2 + (7-2)^2} \text{ or } \sqrt{29}$$

$$AC = \sqrt{(4-2)^2 + (2-8)^2} \text{ or } \sqrt{40}$$

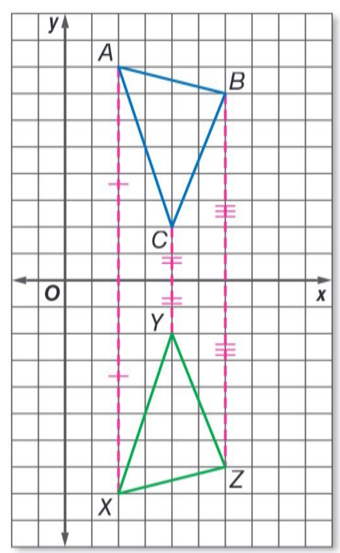
$$XZ = \sqrt{(6-2)^2 + [-7-(-8)]^2} \text{ or } \sqrt{17}$$

$$ZY = \sqrt{(6-4)^2 + [-7-(-2)]^2} \text{ or } \sqrt{29}$$

$$XY = \sqrt{(2-4)^2 + [-8-(-2)]^2} \text{ or } \sqrt{40}$$

Since $AB = XZ$, $BC = ZY$, and $AC = XY$, $\overline{AB} \cong \overline{XZ}$, $\overline{BC} \cong \overline{ZY}$, and $\overline{AC} \cong \overline{XY}$. By SSS, $\triangle ABC \cong \triangle XZY$.

Check Use the definition of a reflection. Use a ruler to measure and compare the segments connecting each vertex and its image to the line of symmetry. These segments are congruent, so the triangles are congruent. ✔



StudyTip

Isometry While an isometry preserves congruence, a *direct isometry* also preserves orientation or order of lettering. An *indirect* or *opposite isometry* changes this order, such as from clockwise to counterclockwise. The reflection shown in Example 3 is an example of an indirect isometry.

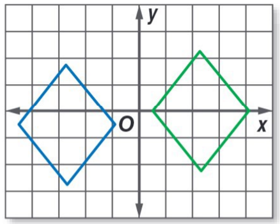
GuidedPractice

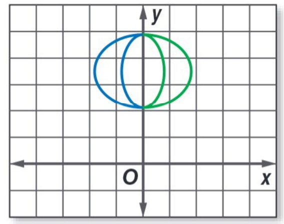
3. Triangle JKL with vertices $J(-2, 2)$, $K(-8, 5)$, and $L(-4, 6)$ is a transformation of $\triangle PQR$ with vertices $P(2, -2)$, $Q(8, -5)$, and $R(4, -6)$. Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 Identify the type of congruence transformation shown as a *reflection, translation,* or *rotation.*

1. 

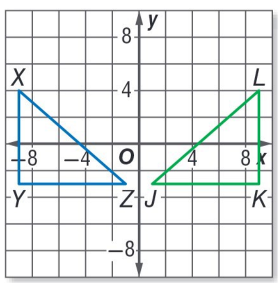
2. 

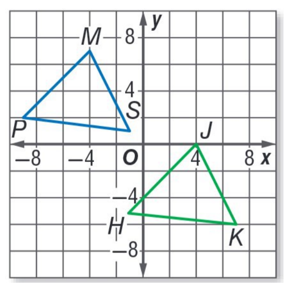
Example 2

3. 

4. 

Example 3 **COORDINATE GEOMETRY** Identify each transformation and verify that it is a congruence transformation.

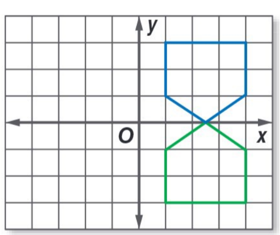
5. 

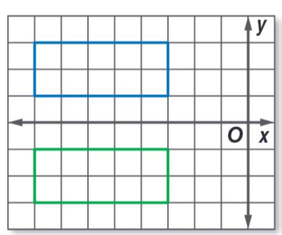
6. 

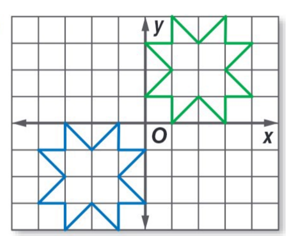
Practice and Problem Solving

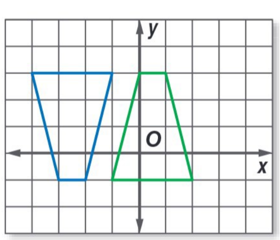
Extra Practice is on page R4.

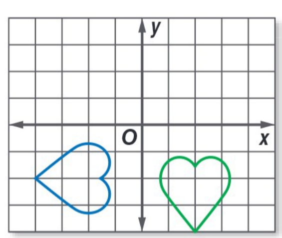
Example 1 CCSS **STRUCTURE** Identify the type of congruence transformation shown as a *reflection, translation,* or *rotation.*

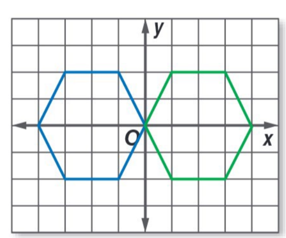
7. 

8. 

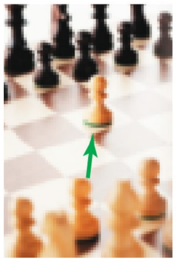
9. 

10. 

11. 

12. 

Example 2 Identify the type of congruence transformation shown in each picture as a *reflection, translation,* or *rotation.*

13. 

14. 

15. 

16. 

Example 3 **COORDINATE GEOMETRY** Graph each pair of triangles with the given vertices. Then, identify the transformation, and verify that it is a congruence transformation.

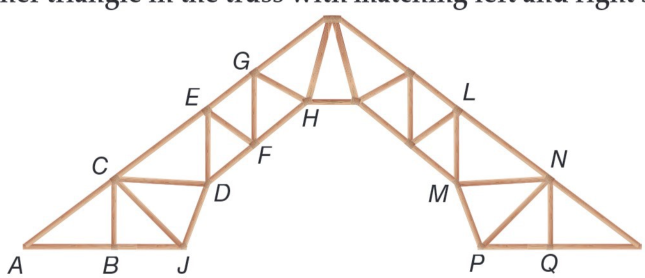
17. $M(-7, -1)$, $P(-7, -7)$, $R(-1, -4)$; $T(7, -1)$, $V(7, -7)$, $S(1, -4)$

18. $A(3, 9)$, $B(3, 7)$, $C(7, 7)$; $S(3, 5)$, $T(3, 3)$, $R(7, 3)$

19. $A(-4, 5)$, $B(0, 2)$, $C(-4, 2)$; $X(-5, -4)$, $Y(-2, 0)$, $Z(-2, -4)$

20. $A(2, 2)$, $B(4, 7)$, $C(6, 2)$; $D(2, -2)$, $F(4, -7)$, $G(6, -2)$

CONSTRUCTION Identify the type of congruence transformation performed on each given triangle to generate the other triangle in the truss with matching left and right sides shown below.



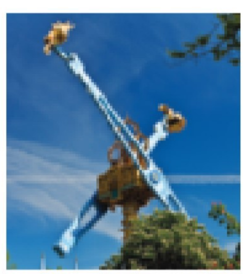
21. $\triangle NMP$ to $\triangle CJD$

22. $\triangle EFD$ to $\triangle GHF$

23. $\triangle CBJ$ to $\triangle NQP$

AMUSEMENT RIDES Identify the type of congruence transformation shown in each picture as a *reflection, translation,* or *rotation.*

24. 

25. 

26. 

27. **SCHOOL** Identify the transformations that are used to open a combination lock on a locker. If appropriate, identify the line of symmetry or center of rotation.

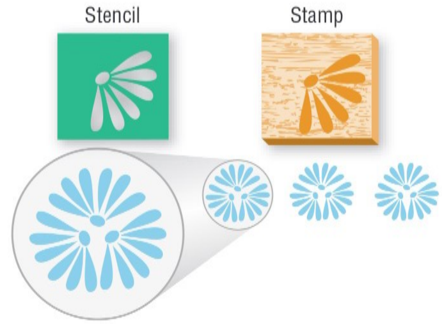
28. CCSS **STRUCTURE** Determine which capital letters of the alphabet have vertical and/or horizontal lines of reflection.

(tl)moodboard/CORBIS, (tr)Robert Harding Picture Library/SuperStock, (cl)Car Culture/Collection Mix: Subjects/Getty Images, (cr)Steve Allen/Brand X Picture/Getty Images, (bl)Kristy-Anne Glubish/Design Pics/CORBIS, (bc)NielsDK/Photolibrary, (br)KFS/age footstock

29. DECORATING Tionne is redecorating her bedroom. She can use stencils or a stamp to create the design shown.

a. If Tionne used the stencil, what type of transformation was used to produce each flower in the design?

b. What type of transformation was used if she used the stamp to produce each flower in the design?



30. MULTIPLE REPRESENTATIONS In this problem, you will investigate the relationship between the ordered pairs of a figure and its translated image.

a. Geometric Draw congruent rectangles $ABCD$ and $WXYZ$ on a coordinate plane.

b. Verbal How do you get from a vertex on $ABCD$ to the corresponding vertex on $WXYZ$ using only horizontal and vertical movement?

c. Tabular Copy the table shown. Use your rectangles to fill in the x-coordinates, the y-coordinates, and the unknown value in the transformation column.

d. Algebraic Function notation $(x, y) \rightarrow (x + a, y + b)$, where a and b are real numbers, represents a mapping from one set of coordinates onto another. Complete the following notation that represents the rule for the translation $ABCD \rightarrow WXYZ$: $(x, y) \rightarrow (x + a, y + b)$.

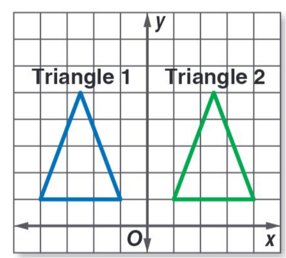
Rectangle *ABCD*	Transformation	Rectangle *WXYZ*
$A(?, ?)$	$(x_1 + ?, y_1 + ?)$	$W(?, ?)$
$B(?, ?)$	$(x_1 + ?, y_1 + ?)$	$X(?, ?)$
$C(?, ?)$	$(x_1 + ?, y_1 + ?)$	$Y(?, ?)$
$D(?, ?)$	$(x_1 + ?, y_1 + ?)$	$Z(?, ?)$

H.O.T. Problems Use Higher-Order Thinking Skills

31. CHALLENGE Use the diagram at the right.

a. Identify two transformations of Triangle 1 that can result in Triangle 2.

b. What must be true of the triangles in order for more than one transformation on a preimage to result in the same image? Explain your reasoning.

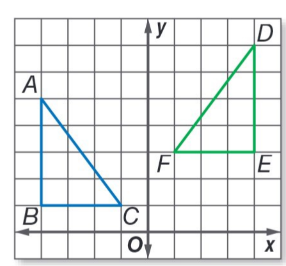


32. CCSS REASONING A *dilation* is another type of transformation. In the diagram, a small paper clip has been dilated to produce a larger paper clip. Explain why dilations are not a congruence transformation.

OPEN ENDED Describe a real-world example of each of the following, other than those given in this lesson.

33. reflection **34.** translation **35.** rotation

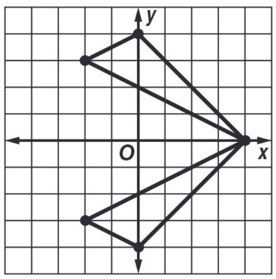
36. WRITING IN MATH In the diagram at the right $\triangle DEF$ is called a *glide reflection* of $\triangle ABC$. Based on the diagram, define a glide reflection. Is a glide reflection a congruence transformation? Include a definition of congruence transformation in your response. Explain your reasoning.



Standardized Test Practice

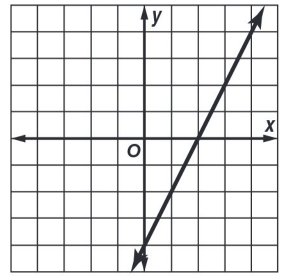
37. SHORT RESPONSE Cindy is shopping for a new desk chair at a store where the desk chairs are 50% off. She also has a coupon for 50% off any one item. Cindy thinks that she can now get the desk chair for free. Is this true? If not, what will be the percent off she will receive with both the sale and the coupon?

38. Identify the congruence transformation shown.



A dilation
B reflection
C rotation
D translation

39. Look at the graph below. What is the slope of the line shown?



F -2
G -1
H 1
J 2

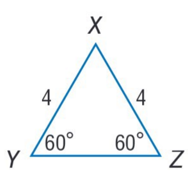
40. SAT/ACT What is the y-intercept of the line determined by the equation $3x - 4 = 12y - 3$?

A -12
B $-\frac{1}{12}$
C $\frac{1}{12}$
D $\frac{1}{4}$
E 12

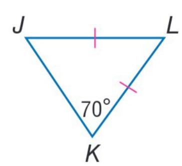
Spiral Review

Find each measure. (Lesson 4-6)

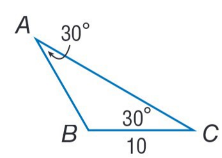
41. YZ



42. $m\angle JLK$



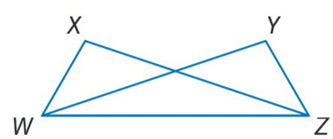
43. AB



44. PROOF Write a paragraph proof. (Lesson 4-5)

Given: $\angle YWZ \cong \angle XZW$ and $\angle YZW \cong \angle XWZ$

Prove: $\triangle WXZ \cong \triangle ZYW$



45. ROLLER COASTERS The sign in front of the Electric Storm roller coaster states that all riders must be at least 54 inches tall to ride. If Andy is 5 feet 8 inches tall, can he ride the Electric Storm? Which law of logic leads you to this conclusion? (Lesson 2-4)

Skills Review

Find the coordinates of the midpoint of a segment with the given endpoints.

46. $A(10, -12), C(5, -6)$

47. $A(13, 14), C(3, 5)$

48. $A(-28, 8), C(-10, 2)$

49. $A(-12, 2), C(-3, 5)$

50. $A(0, 0), C(3, -4)$

51. $A(2, 14), C(0, 5)$

LESSON 4-8 Triangles and Coordinate Proof

Then

- You used coordinate geometry to prove triangle congruence.

Now

1. Position and label triangles for use in coordinate proofs.
2. Write coordinate proofs.

Why?

- A global positioning system (GPS) receives transmissions from satellites that allow the exact location of a car to be determined. The information can be used with navigation software to provide driving directions.

NewVocabulary
coordinate proof

Common Core State Standards

Content Standards
G.CO.10 Prove theorems about triangles.
G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

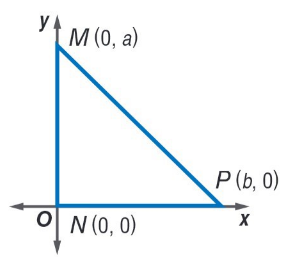
Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.
2 Reason abstractly and quantitatively.

1 Position and Label Triangles

As with global positioning systems, knowing the coordinates of a figure in a coordinate plane allows you to explore its properties and draw conclusions about it. **Coordinate proofs** use figures in the coordinate plane and algebra to prove geometric concepts. The first step in a coordinate proof is placing the figure on the coordinate plane.

Example 1 Position and Label a Triangle

Position and label right triangle MNP on the coordinate plane so that leg $\overline{MN}$ is a units long and leg $\overline{NP}$ is b units long.



- The length(s) of the side(s) that are along the axes will be easier to determine than the length(s) of side(s) that are not along an axis. Since this is a right triangle, two sides can be located on an axis.
- Placing the right angle of the triangle, $\angle N$, at the origin will allow the two legs to be along the x- and y-axes.
- Position the triangle in the first quadrant.
- Since M is on the y-axis, its x-coordinate is 0. Its y-coordinate is a because the leg is a units long.
- Since P is on the x-axis, its y-coordinate is 0. Its x-coordinate is b because the leg is b units long.

GuidedPractice

1. Position and label isosceles triangle JKL on the coordinate plane so that its base $\overline{JL}$ is a units long, vertex K is on the y-axis, and the height of the triangle is b units.

KeyConcept Placing Triangles on Coordinate Plane

Step 1 Use the origin as a vertex or center of the triangle.

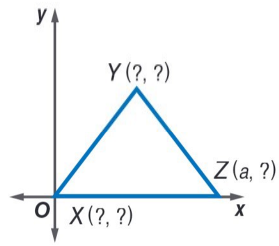
Step 2 Place at least one side of a triangle on an axis.

Step 3 Keep the triangle within the first quadrant if possible.

Step 4 Use coordinates that make computations as simple as possible.

Example 2 Identify Missing Coordinates

Name the missing coordinates of isosceles triangle XYZ.



Vertex X is positioned at the origin; its coordinates are $(0, 0)$.

Vertex Z is on the x-axis, so its y-coordinate is 0. The coordinates of vertex Z are $(a, 0)$.

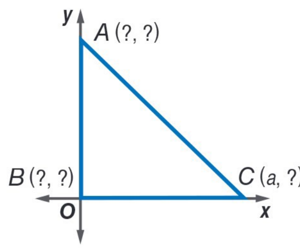
$\triangle XYZ$ is isosceles, so using a vertical segment from Y to to the x-axis and the Hypotenuse-Leg Theorem shows that the x-coordinate of Y is halfway between 0 and a or $\frac{a}{2}$. We cannot write the y-coordinate in terms of a, so call it b. The coordinates of point Y are $\left(\frac{a}{2}, b\right)$.

Study Tip

Right Angle The intersection of the x- and y-axis forms a right angle, so it is a convenient place to locate the right angle of a figure such as a right triangle.

Guided Practice

2. Name the missing coordinates of isosceles right triangle ABC.



2 Write Coordinate Proofs

After a triangle is placed on the coordinate plane and labeled, we can use coordinate proofs to verify properties and to prove theorems.

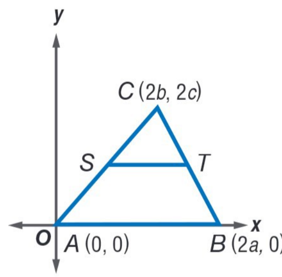
Study Tip

Coordinate Proof The guidelines and methods used in this lesson apply to all polygons, not just triangles.

Example 3 Write a Coordinate Proof

Write a coordinate proof to show that a line segment joining the midpoints of two sides of a triangle is parallel to the third side.

Place a vertex at the origin and label it A. Use coordinates that are multiples of 2 because the Midpoint Formula involves dividing the sum of the coordinates by 2.



Given: $\triangle ABC$
S is the midpoint of $\overline{AC}$.
T is the midpoint of $\overline{BC}$.

Prove: $\overline{ST} \parallel \overline{AB}$

Proof:

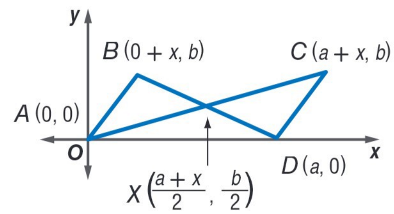
By the Midpoint Formula, the coordinates of S are $\left(\frac{2b+0}{2}, \frac{2c+0}{2}\right)$ or (b, c) and the coordinates of T are $\left(\frac{2a+2b}{2}, \frac{0+2c}{2}\right)$ or $(a + b, c)$.

By the Slope Formula, the slope of $\overline{ST}$ is $\frac{c-c}{a+b-b}$ or 0 and the slope of $\overline{AB}$ is $\frac{0-0}{2a-0}$ or 0.

Since $\overline{ST}$ and $\overline{AB}$ have the same slope, $\overline{ST} \parallel \overline{AB}$.

GuidedPractice

3. Write a coordinate proof to show that $\triangle ABX \cong \triangle CDX$.



Real-WorldLink

More than 50 ships and 20 airplanes have mysteriously disappeared from a section of the North Atlantic Ocean off of North America commonly referred to as the Bermuda Triangle.

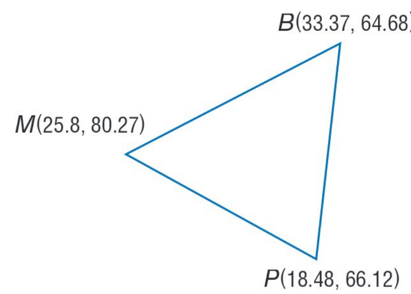
Source: *Encyclopaedia Britannica*

The techniques used for coordinate proofs can be used to solve real-world problems.

Real-World Example 4 Classify Triangles

GEOGRAPHY The Bermuda Triangle is a region formed by Miami, Florida, San Jose, Puerto Rico, and Bermuda. The approximate coordinates of each location, respectively, are 25.8°N 80.27°W, 18.48°N 66.12°W, and 33.37°N 64.68°W. Write a coordinate proof to prove that the Bermuda Triangle is scalene.

The first step is to label the coordinates of each location. Let M represent Miami, B represent Bermuda, and P represent Puerto Rico.



If no two sides of $\triangle MPB$ are congruent, then the Bermuda Triangle is scalene. Use the Distance Formula and a calculator to find the distance between each location.

$$MB = \sqrt{(33.37 - 25.8)^2 + (64.68 - 80.27)^2}$$
$$\approx 17.33$$

$$MP = \sqrt{(25.8 - 18.48)^2 + (80.27 - 66.12)^2}$$
$$\approx 15.93$$

$$PB = \sqrt{(33.37 - 18.48)^2 + (64.68 - 66.12)^2}$$
$$\approx 14.96$$

Since each side is a different length, $\triangle MPB$ is scalene. Therefore, the Bermuda Triangle is scalene.

GuidedPractice

4. **GEOGRAPHY** In 2006, a group of art museums collaborated to form the West Texas Triangle to promote their collections. This region is formed by the cities of Odessa, Albany, and San Angelo. The approximate coordinates of each location, respectively, are 31.9°N 102.3°W, 32.7°N 99.3°W, and 31.4°N 100.5°W. Write a coordinate proof to prove that the West Texas Triangle is approximately isosceles.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

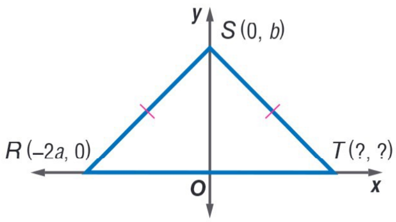
Example 1 **Position and label each triangle on the coordinate plane.**

1. right $\triangle ABC$ with legs $\overline{AC}$ and $\overline{AB}$ so that $\overline{AC}$ is $2a$ units long and leg $\overline{AB}$ is $2b$ units long

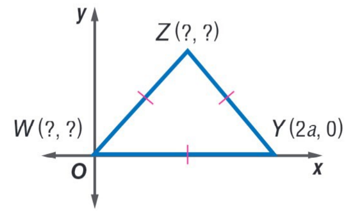
2. isosceles $\triangle FGH$ with base $\overline{FG}$ that is $2a$ units long

Example 2 **Name the missing coordinate(s) of each triangle.**

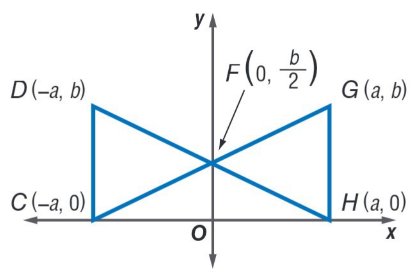
3.



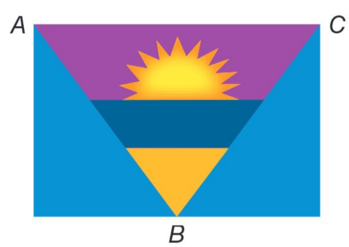
4.



Example 3 **5.** CCSS **ARGUMENTS** Write a coordinate proof to show that $\triangle FGH \cong \triangle FDC$.



Example 4 **6.** **FLAGS** Write a coordinate proof to prove that the large triangle in the center of the flag is isosceles. The dimensions of the flag are 4 feet by 6 feet and point B of the triangle bisects the bottom of the flag.



Practice and Problem Solving

Extra Practice is on page R4.

Example 1 **Position and label each triangle on the coordinate plane.**

7. isosceles $\triangle ABC$ with base $\overline{AB}$ that is a units long

8. right $\triangle XYZ$ with hypotenuse $\overline{YZ}$, the length of $\overline{XY}$ is b units long, and the length of $\overline{XZ}$ is three times the length of $\overline{XY}$

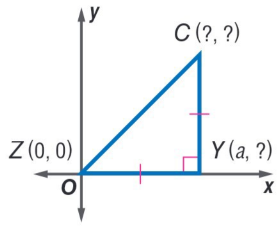
9 isosceles right $\triangle RST$ with hypotenuse $\overline{RS}$ and legs $3a$ units long

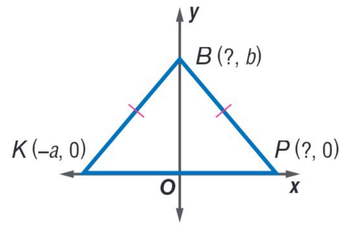
10. right $\triangle JKL$ with legs $\overline{JK}$ and $\overline{KL}$ so that $\overline{JK}$ is a units long and leg $\overline{KL}$ is $4b$ units long

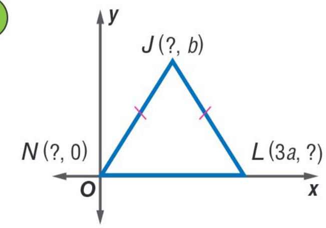
11. equilateral $\triangle GHJ$ with sides $\frac{1}{2}a$ units long

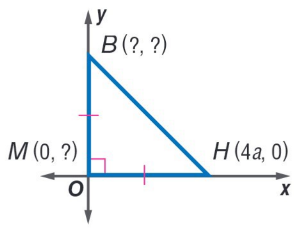
12. equilateral $\triangle DEF$ with sides $4b$ units long

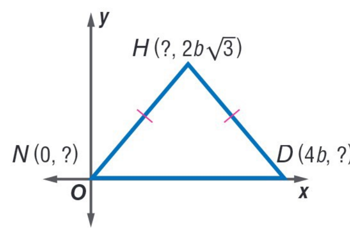
Example 2 **Name the missing coordinate(s) of each triangle.**

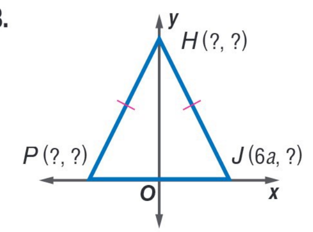
13. 

14. 

15. 

16. 

17. 

18. 

Example 3 CCSS **ARGUMENTS** **Write a coordinate proof for each statement.**

19. The segments joining the base vertices to the midpoints of the legs of an isosceles triangle are congruent.

20. The three segments joining the midpoints of the sides of an isosceles triangle form another isosceles triangle.

Example 4 **PROOF** **Write a coordinate proof for each statement.**

21. The measure of the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.

22. If a line segment joins the midpoints of two sides of a triangle, then its length is equal to one half the length of the third side.

23. **RESEARCH TRIANGLE** The cities of Raleigh, Durham, and Chapel Hill, North Carolina, form what is known as the Research Triangle. The approximate latitude and longitude of Raleigh are 35.82°N 78.64°W, of Durham are 35.99°N 78.91°W, and of Chapel Hill are 35.92°N 79.04°W. Show that the triangle formed by these three cities is scalene.

24. **PARTY PLANNING** Three friends live in houses with backyards adjacent to a neighborhood bike path. They decide to have a round-robin party using their three homes, inviting their friends to start at one house and then move to each of the other two. If one friend's house is centered at the origin, then the location of the other homes are (5, 12) and (13, 0). Write a coordinate proof to prove that the triangle formed by these three homes is isosceles.

Draw $\triangle XYZ$ and find the slope of each side of the triangle. Determine whether the triangle is a right triangle. Explain.

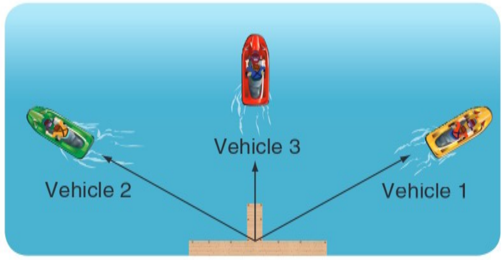
25. $X(0, 0)$, $Y(2h, 2h)$, $Z(4h, 0)$

26. $X(0, 0)$, $Y(1, h)$, $Z(2h, 0)$

27. **CAMPING** Two families set up tents at a state park. If the ranger's station is located at (0, 0), and the locations of the tents are (0, 25) and (12, 9), write a coordinate proof to prove that the figure formed by the locations of the ranger's station and the two tents is a right triangle.

28. **PROOF** Write a coordinate proof to prove that $\triangle ABC$ is an isosceles triangle if the vertices are $A(0, 0)$, $B(a, b)$, and $C(2a, 0)$.

29 **WATER SPORTS** Three personal watercraft vehicles launch from the same dock. The first vehicle leaves the dock traveling due northeast, while the second vehicle travels due northwest. Meanwhile, the third vehicle leaves the dock traveling due north.



The first and second vehicles stop about 300 yards from the dock, while the third stops about 212 yards from the dock.

a. If the dock is located at (0, 0), sketch a graph to represent this situation. What is the equation of the line along which the first vehicle lies? What is the equation of the line along which the second vehicle lies? Explain your reasoning.

b. Write a coordinate proof to prove that the dock, the first vehicle, and the second vehicle form an isosceles right triangle.

c. Find the coordinates of the locations of all three watercrafts. Explain your reasoning.

d. Write a coordinate proof to prove that the positions of all three watercrafts are approximately collinear and that the third watercraft is at the midpoint between the other two.

H.O.T. Problems Use Higher-Order Thinking Skills

30. **REASONING** The midpoints of the sides of a triangle are located at $(a, 0)$, $(2a, b)$ and (a, b). If one vertex is located at the origin, what are the coordinates of the other vertices? Explain your reasoning.

CHALLENGE **Find the coordinates of point L so $\triangle JKL$ is the indicated type of triangle. Point J has coordinates (0, 0) and point K has coordinates $(2a, 2b)$.**

31. scalene triangle **32.** right triangle **33.** isosceles triangle

34. **OPEN ENDED** Draw an isosceles right triangle on the coordinate plane so that the midpoint of its hypotenuse is the origin. Label the coordinates of each vertex.

35. **CHALLENGE** Use a coordinate proof to show that if you add n units to each x-coordinate of the vertices of a triangle and m to each y-coordinate, the resulting figure is congruent to the original triangle.

36. CCSS **REASONING** A triangle has vertex coordinates (0, 0) and $(a, 0)$. If the coordinates of the third vertex are in terms of a, and the triangle is isosceles, identify the coordinates and position the triangle on the coordinate plane.

37. **WRITING IN MATH** Explain why following each guideline below for placing a triangle on the coordinate plane is helpful in proving coordinate proofs.

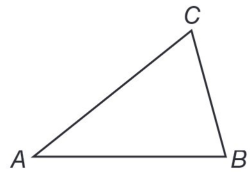
a. Use the origin as a vertex of the triangle.

b. Place at least one side of the triangle on the x- or y-axis.

c. Keep the triangle within the first quadrant if possible.

Standardized Test Practice

38. GRIDDED RESPONSE In the figure below, $m\angle B = 76$. The measure of $\angle A$ is half the measure of $\angle B$. What is $m\angle C$?



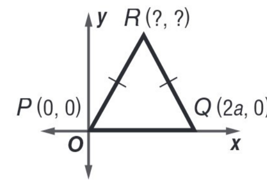
39. ALGEBRA What is the x-coordinate of the solution to the system of equations shown below?

$$\begin{cases} 2x - 3y = 3 \\ -4x + 2y = -18 \end{cases}$$

A -6 **C** 3

B -3 **D** 6

40. What are the coordinates of point R in the triangle?



F $\left(\frac{a}{2}, a\right)$ **H** $\left(\frac{b}{2}, a\right)$

G (a, b) **J** $\left(\frac{b}{2}, \frac{a}{2}\right)$

41. SAT/ACT For all x,
$17x^5 + 3x^2 + 2 - (-4x^5 + 3x^3 - 2) =$

A $13x^5 + 3x^3 + 3x^2$

B $13x^5 + 6x^2 + 4$

C $21x^5 - 3x^3 + 3x^2 + 4$

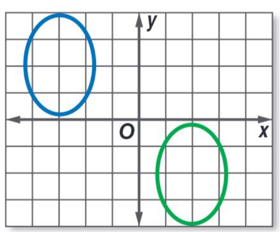
D $21x^5 + 3x^2 + 3x^3$

E $21x^5 + 3x^3 + 3x^2 + 4$

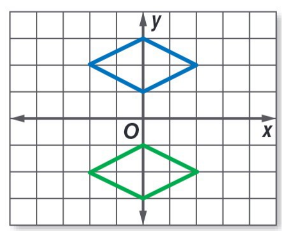
Spiral Review

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*. (Lesson 4-7)

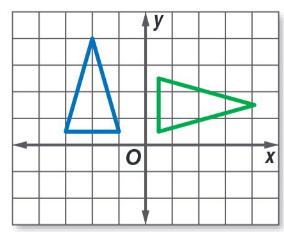
42.



43.



44.

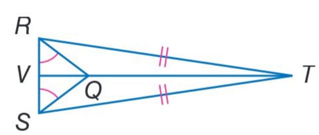


Refer to the figure at the right. (Lesson 4-6)

45. Name two congruent angles.

46. Name two congruent segments.

47. Name a pair of congruent triangles.



48. RAMPS The Americans with Disabilities Act requires that wheelchair ramps have at least a 12-inch run for each rise of 1 inch. (Lesson 3-3)

a. Determine the slope represented by this requirement.

b. The maximum length that the law allows for a ramp is 30 feet. How many inches tall is the highest point of this ramp?

Skills Review

Find the distance between each pair of points. Round to the nearest tenth.

49. $X(5, 4)$ and $Y(2, 1)$

50. $A(1, 5)$ and $B(-2, -3)$

51. $J(-2, 6)$ and $K(1, 4)$

CHAPTER 4

Study Guide and Review

Study Guide

KeyConcepts

Classifying Triangles (Lesson 4-1)

- Triangles can be classified by their angles as acute, obtuse, or right, and by their sides as scalene, isosceles, or equilateral.

Angles of Triangles (Lesson 4-2)

- The measure of an exterior angle is equal to the sum of its two remote interior angles.

Congruent Triangles (Lesson 4-3 through 4-5)

- SSS: If all of the corresponding sides of two triangles are congruent, then the triangles are congruent.
- SAS: If two pairs of corresponding sides of two triangles and the included angles are congruent, then the triangles are congruent.
- ASA: If two pairs of corresponding angles of two triangles and the included sides are congruent, then the triangles are congruent.
- AAS: If two pairs of corresponding angles of two triangles are congruent, and a corresponding pair of nonincluded sides is congruent, then the triangles are congruent.

Isosceles and Equilateral Triangles (Lesson 4-6)

- The base angles of an isosceles triangle are congruent and a triangle is equilateral if it is equiangular.

Transformations and Coordinate Proofs
(Lessons 4-7 and 4-8)

- In a congruence transformation, the position of the image may differ from the preimage, but the two figures remain congruent.
- Coordinate proofs use algebra to prove geometric concepts.

FOLDABLES® StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.

KeyVocabulary

acute triangle (p. 237)
auxiliary line (p. 246)
base angles (p. 285)
congruence transformation (p. 296)
congruent polygons (p. 255)
coordinate proof (p. 303)
corollary (p. 249)
corresponding parts (p. 255)
equiangular triangle (p. 237)
equilateral triangle (p. 238)
exterior angle (p. 248)
flow proof (p. 248)
included angle (p. 266)
included side (p. 275)
isosceles triangle (p. 238)
obtuse triangle (p. 237)
reflection (p. 296)
remote interior angles (p. 248)
right triangle (p. 237)
rotation (p. 296)
scalene triangle (p. 238)
translation (p. 296)
vertex angle (p. 285)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

1. An equiangular triangle is also an example of an acute triangle.
2. A triangle with an angle that measures greater than 90° is a right triangle.
3. An equilateral triangle is always equiangular.
4. A scalene triangle has at least two congruent sides.
5. The vertex angles of an isosceles triangle are congruent.
6. An included side is the side located between two consecutive angles of a polygon.
7. The three types of congruence transformations are rotation, reflection, and translation.
8. A rotation moves all points of a figure the same distance and in the same direction.
9. A flow proof uses figures in the coordinate plane and algebra to prove geometric concepts.
10. The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

Lesson-by-Lesson Review

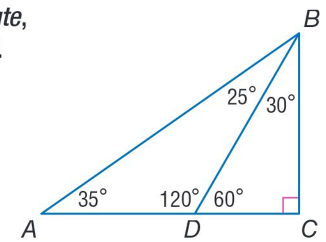
4-1 Classifying Triangles

Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

11. $\triangle ADB$

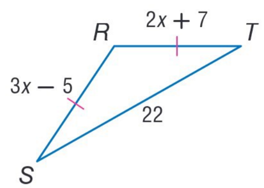
12. $\triangle BCD$

13. $\triangle ABC$

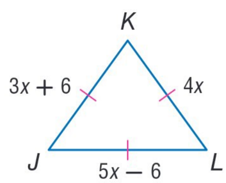


ALGEBRA Find x and the measures of the unknown sides of each triangle.

14.



15.

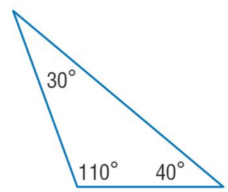


16. **MAPS** The distance from Chicago to Cleveland to Cincinnati and back to Chicago is 900 miles. The distance from Chicago to Cleveland is 50 miles more than the distance from Cincinnati to Chicago, and the distance from Cleveland to Cincinnati is 50 miles less than the distance from Cincinnati to Chicago. Find each distance and classify the triangle formed by the three cities.

Example 1

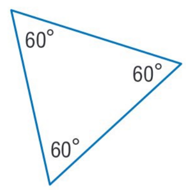
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

a.



Since the triangle has one obtuse angle, it is an obtuse triangle.

b.



The triangle has three acute angles that are all equal. It is an equiangular triangle.

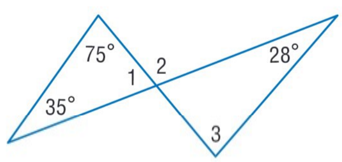
4-2 Angles of Triangles

Find the measure of each numbered angle.

17. $\angle 1$

18. $\angle 2$

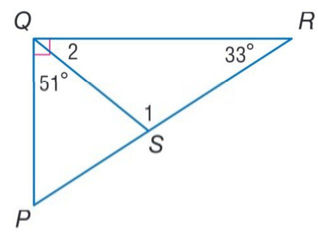
19. $\angle 3$



20. **HOUSES** The roof support on Lamar's house is in the shape of an isosceles triangle with base angles of 38°. Find x.

Example 2

Find the measure of each numbered angle.



$m\angle 2 + m\angle PQS = 90$

$m\angle 2 + 51 = 90$ — Substitution

$m\angle 2 = 39$ — Subtract 51 from each side.

$m\angle 1 + m\angle 2 + 33 = 180$ — Triangle Sum Theorem

$m\angle 1 + 39 + 33 = 180$ — Substitution

$m\angle 1 + 72 = 180$ — Simplify.

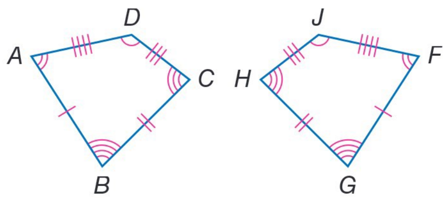
$m\angle 1 = 108$ — Subtract.

Study Guide and Review *Continued*

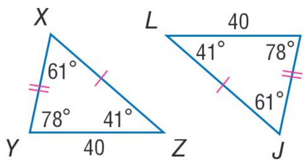
4-3 Congruent Triangles

Show that the polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

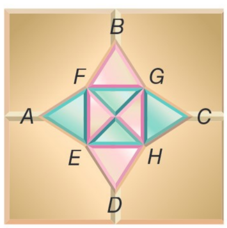
21.



22.

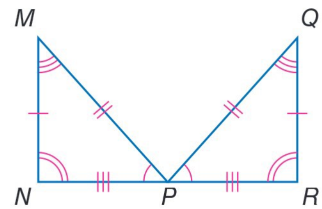


23. **MOSAIC TILING** A section of a mosaic tiling is shown. Name the triangles that appear to be congruent.



Example 3

Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a congruence statement.



Angles: $\angle N \cong \angle R$, $\angle M \cong \angle Q$, $\angle MPN \cong \angle QPR$

Sides: $\overline{MN} \cong \overline{QR}$, $\overline{MP} \cong \overline{QP}$, $\overline{NP} \cong \overline{RP}$

All corresponding parts of the two triangles are congruent. Therefore, $\triangle MNP \cong \triangle QRP$.

4-4 Proving Triangles Congruent—SSS, SAS

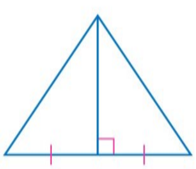
Determine whether $\triangle ABC \cong \triangle XYZ$. Explain.

24. $A(5, 2)$, $B(1, 5)$, $C(0, 0)$, $X(-3, 3)$, $Y(-7, 6)$, $Z(-8, 1)$

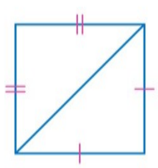
25. $A(3, -1)$, $B(3, 7)$, $C(7, 7)$, $X(-7, 0)$, $Y(-7, 4)$, $Z(1, 4)$

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

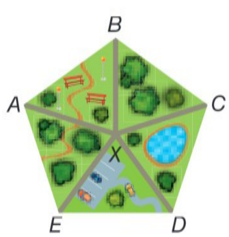
26.



27.

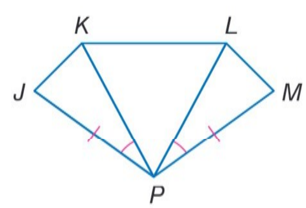


28. **PARKS** The diagram shows a park in the shape of a pentagon with five sidewalks of equal length leading to a central point. If all the angles at the central point have the same measure, how could you prove that $\triangle ABX \cong \triangle DCX$?



Example 4

Write a two-column proof.



Given: $\triangle KPL$ is equilateral.

$\overline{JP} \cong \overline{MP}$,
$\angle JPK \cong \angle MPL$

Prove: $\triangle JPK \cong \triangle MPL$

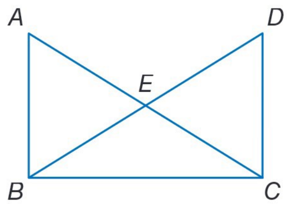
Statements	Reasons
1. $\triangle KPL$ is equilateral.	**1.** Given
2. $\overline{PK} \cong \overline{PL}$	**2.** Def. of Equilateral $\triangle$
3. $\overline{JP} \cong \overline{MP}$	**3.** Given
4. $\angle JPK \cong \angle MPL$	**4.** Given
5. $\triangle JPK \cong \triangle MPL$	**5.** SAS

4-5 Proving Triangles Congruent—ASA, AAS

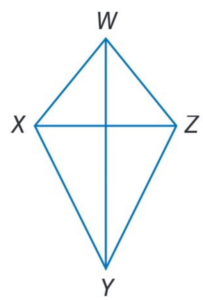
Write a two-column proof.

29. Given: $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$

Prove: $\triangle ABE \cong \triangle CDE$



30. **KITES** Denise's kite is shown in the figure at the right. Given that $\overline{WY}$ bisects both $\angle XWZ$ and $\angle XYZ$, prove that $\triangle WXY \cong \triangle WZY$.

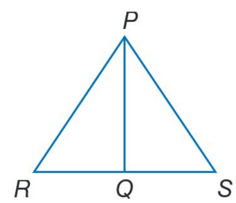


Example 5

Write a flow proof.

Given: $\overline{PQ}$ bisects $\angle RPS$.
$\angle R \cong \angle S$

Prove: $\triangle RPQ \cong \triangle SPQ$



Flow Proof:

$\overline{PQ}$ bisects $\angle RPS$. — **Given** → $\angle RPQ \cong \angle SPQ$ — Def. of $\angle$ Bisector

$\angle R \cong \angle S$ — **Given**

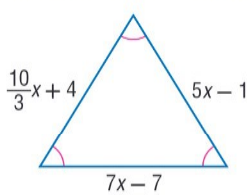
$\overline{PQ} \cong \overline{PQ}$ — **Reflexive**

→ $\triangle RPQ \cong \triangle SPQ$ — AAS

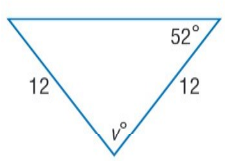
4-6 Isosceles and Equilateral Triangles

Find the value of each variable.

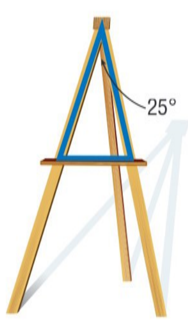
31.



32.

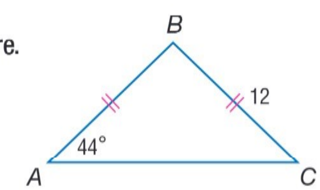


33. **PAINTING** Pam is painting using a wooden easel. The support bar on the easel forms an isosceles triangle with the two front supports. According to the figure below, what are the measures of the base angles of the triangle?



Example 6

Find each measure.



a. $m\angle B$

Since $AB = BC$, $\overline{AB} \cong \overline{BC}$. By the Isosceles Triangle Theorem, base angles A and C are congruent, so $m\angle A = m\angle C$. Use the Triangle Sum Theorem to write and solve an equation to find $m\angle B$.

$m\angle A + m\angle B + m\angle C = 180$	$\triangle$ Sum Theorem
$44 + m\angle B + 44 = 180$	$m\angle A = m\angle C = 44$
$88 + m\angle B = 180$	Simplify.
$m\angle B = 92$	Subtract.

b. AB

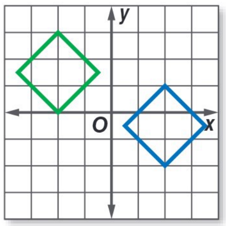
$AB = BC$, so $\triangle ABC$ is isosceles. Since $BC = 12$, $AB = 12$ by substitution.

Study Guide and Review *Continued*

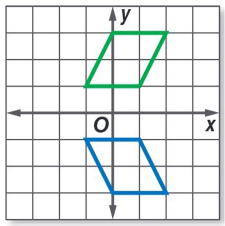
4-7 Congruence Transformations

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

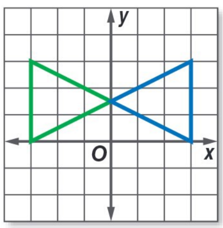
34.



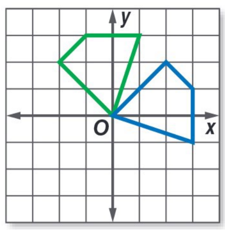
35.



36.



37.



38. Triangle ABC with vertices $A(1, 1)$, $B(2, 3)$, and $C(3, -1)$ is a transformation of $\triangle MNO$ with vertices $M(-1, 1)$, $N(-2, 3)$, and $O(-3, -1)$. Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.

Example 7

Triangle RST with vertices $R(4, 1)$, $S(2, 5)$, and $T(-1, 0)$ is a transformation of $\triangle CDF$ with vertices $C(1, -3)$, $D(-1, 1)$, and $F(-4, -4)$. Identify the transformation and verify that it is a congruence transformation.

Graph each figure. The transformation appears to be a translation. Find the lengths of the sides of each triangle.

$$RS = \sqrt{(4-2)^2 + (1-5)^2} \text{ or } \sqrt{20}$$
$$TS = \sqrt{(-1-2)^2 + (0-5)^2} \text{ or } \sqrt{34}$$
$$RT = \sqrt{(-1-4)^2 + (0-1)^2} \text{ or } \sqrt{26}$$
$$CD = \sqrt{(-1-1)^2 + [1-(-3)]^2} \text{ or } \sqrt{20}$$
$$DF = \sqrt{[-4-(-1)]^2 + (-4-1)^2} \text{ or } \sqrt{34}$$
$$CF = \sqrt{(-4-1)^2 + [-4-(-3)]^2} \text{ or } \sqrt{26}$$

Since each vertex of $\triangle CDF$ has undergone a transformation 3 units to the right and 4 units up, this is a translation.

Since $RS = CD$, $TS = DF$, and $RT = CF$, $\overline{RS} \cong \overline{CD}$, $\overline{TS} \cong \overline{DF}$, and $\overline{RT} \cong \overline{CF}$. By SSS, $\triangle RST \cong \triangle CDF$.

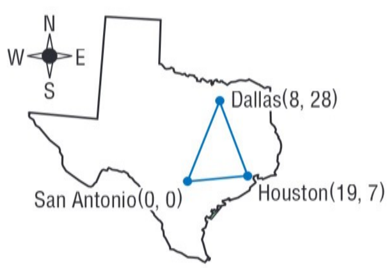
4-8 Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane.

39. right $\triangle MNO$ with right angle at point M and legs of lengths a and $2a$.

40. isosceles $\triangle WXY$ with height h and base $\overline{WY}$ with length $2a$.

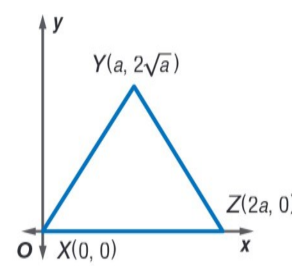
41. **GEOGRAPHY** Jorge plotted the cities of Dallas, San Antonio, and Houston as shown. Write a coordinate proof to show that the triangle formed by these cities is scalene.



Example 8

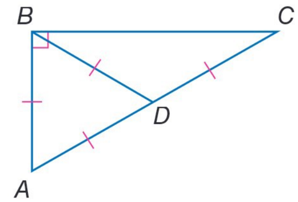
Position and label an equilateral triangle $\triangle XYZ$ with side lengths of $2a$.

- Use the origin for one of the three vertices of the triangle.
- Place one side of the triangle along the positive side of the x-axis.
- The third point should be located above the midpoint of the base of the triangle.



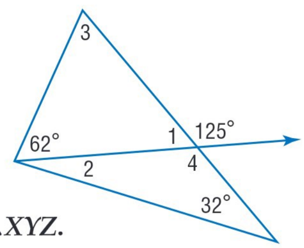
Practice Test

Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.



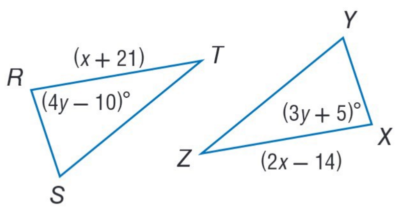
1. $\triangle ABD$
2. $\triangle ABC$
3. $\triangle BDC$

Find the measure of each numbered angle.



4. $\angle 1$
5. $\angle 2$
6. $\angle 3$
7. $\angle 4$

In the diagram, $\triangle RST \cong \triangle XYZ$.

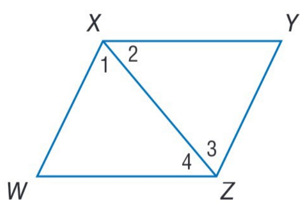


8. Find x.
9. Find y.

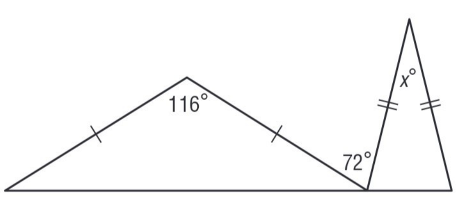
10. **PROOF** Write a flow proof.

Given: $\overline{XY} \parallel \overline{WZ}$ and $\overline{XW} \parallel \overline{YZ}$

Prove: $\triangle XWZ \cong \triangle ZYX$



11. **MULTIPLE CHOICE** Find x.

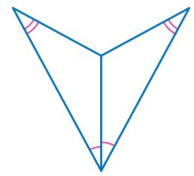


A 36
B 32
C 28
D 22

12. Determine whether $\triangle TJD \cong \triangle SEK$ given $T(-4, -2)$, $J(0, 5)$, $D(1, -1)$, $S(-1, 3)$, $E(3, 10)$, and $K(4, 4)$. Explain.

Determine which postulate or theorem can be used to prove each pair of triangles congruent. If it is not possible to prove them congruent, write *not possible*.

13.



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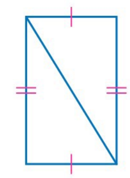
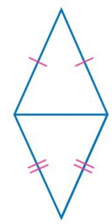
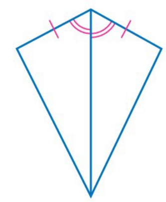


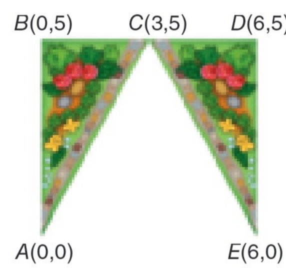
15.



16.

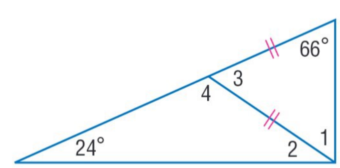


17. **LANDSCAPING** Angie has laid out a design for a garden consisting of two triangular areas as shown below. The points are $A(0, 0)$, $B(0, 5)$, $C(3, 5)$, $D(6, 5)$, and $E(6, 0)$. Name the type of congruence transformation for the preimage $\triangle ABC$ to $\triangle EDC$.



Find the measure of each numbered angle.

18. $\angle 1$
19. $\angle 2$



20. **PROOF** $\triangle ABC$ is a right isosceles triangle with hypotenuse $\overline{AB}$. M is the midpoint of $\overline{AB}$. Write a coordinate proof to show that $\overline{CM}$ is perpendicular to $\overline{AB}$.

CHAPTER 4 Preparing for Standardized Tests

Short-Answer Questions

Short-answer questions require you to provide a solution to the problem, along with a method, explanation, and/or justification used to arrive at the solution.

Short-answer questions are typically graded using a **rubric**, or a scoring guide.

The following is an example of a short-answer question scoring rubric.

Scoring Rubric		
	Criteria	Score
Full Credit	The answer is correct and a full explanation is provided that shows each step.	2
Partial Credit	• The answer is correct, but the explanation is incomplete. • The answer is incorrect, but the explanation is correct.	1 1
No Credit	Either an answer is not provided or the answer does not make sense.	0

Strategies for Solving Short-Answer Questions

Step 1

Read the problem to gain an understanding of what you are trying to solve.

- Identify relevant facts.
- Look for key words and mathematical terms.

Step 2

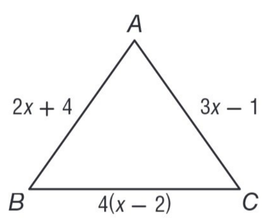
Make a plan and solve the problem.

- Explain your reasoning or state your approach to solving the problem.
- Show all of your work or steps.
- Check your answer if time permits.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

Triangle ABC is an isosceles triangle with base $\overline{BC}$. What is the perimeter of the triangle?



Rob Gage/Taxi/Getty Images

Read the problem carefully. You are told that $\triangle ABC$ is isosceles with base $\overline{BC}$. You are asked to find the perimeter of the triangle.

Make a plan and solve the problem.

The legs of an isosceles triangle are congruent. So, $\overline{AB} \cong \overline{AC}$ or $AB = AC$. Solve for x.

$$\begin{aligned} AB &= AC \\ 2x + 4 &= 3x - 1 \\ 2x - 3x &= -1 - 4 \\ -x &= -5 \\ x &= 5 \end{aligned}$$

Next, find the length of each side.

$AB = 2(5) + 4 = 14$ units
$AC = 3(5) - 1 = 14$ units
$BC = 4(5 - 2) = 12$ units

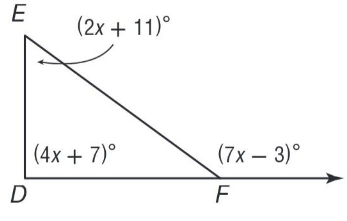
The perimeter of $\triangle ABC$ is $14 + 14 + 12 = 40$ units.

The steps, calculations, and reasoning are clearly stated. The student also arrives at the correct answer. So, this response is worth the full 2 points.

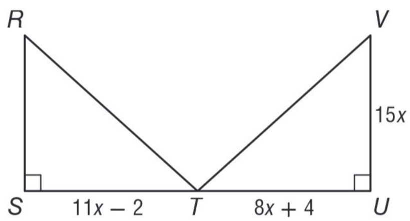
Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. Classify $\triangle DEF$ according to its angle measures.

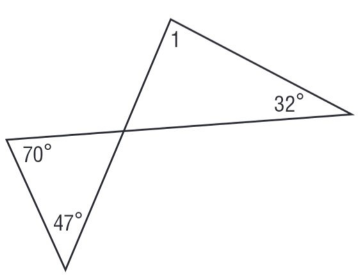


2. In the figure below, $\triangle RST \cong \triangle VUT$. What is the area of $\triangle RST$?



3. A farmer needs to make a 48-square-foot rectangular enclosure for chickens. He wants to save money by purchasing the least amount of fencing possible to enclose the area. What whole-number dimensions will require the least amount of fencing?

4. What is $m\angle 1$ in degrees?

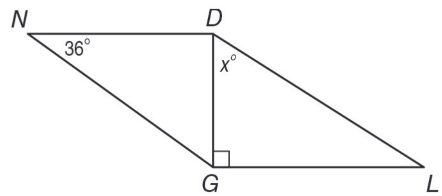


5. Write an equation of the line containing the points $(2, 4)$ and $(0, -2)$.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. **GRIDDED RESPONSE** In the figure below, $\triangle NDG \cong \triangle LGD$. What is the value of x?

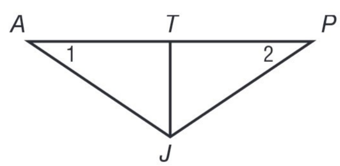


9. **GRIDDED RESPONSE** Suppose line ℓ contains points A, B, and C. If $AB = 7$ inches, $AC = 32$ inches, and point B is between points A and C, what is the length of $\overline{BC}$? Express your answer in inches.

10. Write the converse of the statement.

If you are the winner, then I am the loser.

11. Use the figure and the given information below.

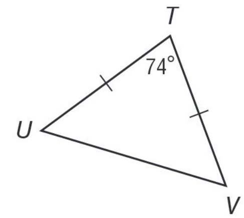


Given: $\overline{JT} \perp \overline{AP}$
$\angle 1 \cong \angle 2$

Which congruence theorem could you use to prove $\triangle PTJ \cong \triangle ATJ$ with only the information given? Explain.

12. Write an equation in slope intercept form for the line which goes through the points (0, 3) and (4, −5).

13. **GRIDDED RESPONSE** Find $m\angle TUV$ in the figure.



14. Suppose two sides of triangle ABC are congruent to two sides of triangle MNO. Also, suppose one of the nonincluded angles of $\triangle ABC$ is congruent to one of the nonincluded angles of $\triangle MNO$. Are the triangles congruent? If so, write a paragraph proof showing the congruence. If not, sketch a counterexample.

Extended Response

Record your answers on a sheet of paper. Show your work.

15. Use a coordinate grid to write a coordinate proof of the following statement.

If the vertices of a triangle are $A(0, 0)$, $B(2a, b)$, and $C(4a, 0)$, then the triangle is isosceles.

a. Plot the vertices on a coordinate grid to model the problem.

b. Use the Distance Formula to write an expression for AB.

c. Use the Distance Formula to write an expression for BC.

d. Use your results from parts **b** and **c** to draw a conclusion about $\triangle ABC$.

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Go to Lesson...	3-2	4-7	4-1	4-3	1-7	4-2	4-6	4-3	1-2	2-3	4-5	3-4	4-6	4-4	4-8